

# Sampling in Exploration: Theory and Practice

J W MERKS<sup>1</sup>

## ABSTRACT

Metal grades and contents of ore deposits should be measured in an unbiased manner, and with a degree of precision that reduces to an acceptable level the risk to put into production inviable prospects. The additive property of variances for metal grades and contents is used to assess the accuracy and precision characteristics of this measurement process. Each stage in the process of extracting different types of samples from ore deposits, preparing test samples from drill core sections, drill chips or crushed ore, and collecting and assaying test portions, contributes a variance component to the intrinsic variability of a metal's distribution in its matrix. Deducting such extrinsic variances results in more precise estimates for metal grades and contents of elementary units. The cumulative metal content for a set of elementary units of in-situ ore, and the sum of the variances for metal contents of all units in the set, can be used to calculate confidence intervals and confidence ranges at different probability levels as measures for risk.

## INTRODUCTION

The development of a prospect into a producing mine involves scores of uncertainties, not the least of which is the measurement process applied to estimate the content of an ore deposit. Uncertainties in this measurement process can be divided into random variations that are intrinsic to each stage, and systematic errors that should be avoided if possible at affordable cost, or taken into account by applying a bias correction factor.

The objective of exploration projects is to estimate metal grades and contents of ore deposits in an unbiased manner, and with a degree of precision that reduces to a minimum the risk to put into production an inviable prospect. The accuracy and precision characteristics of the measurement process determine the magnitude of this risk.

The concepts of accuracy and precision have received a great deal of attention and scrutiny from ISO Technical Committees that deal with sampling methods. The following definitions reflect that accuracy and precision are descriptive terms rather than quantitative measures:

*Accuracy: A generic term that implies closeness of agreement between a measurement and its unknown true value.*

Accuracy is an abstract concept that cannot be quantified. By contrast, a lack of accuracy can be measured and expressed in terms of a systematic error or bias. Webster's New Collegiate Dictionary defines 'accuracy' as 'free from error' which implies that unbiased measurements are accurate by definition.

*Precision: A generic term that refers to the magnitude of random variations in a measurement process.*

Precision too, when used without a quantitative measure, is an abstract concept. For example, the precision is low or poor, or the degree of precision is high or excellent, are valid albeit ambiguous and vacant statements. The variance is the most basic quantitative measure for precision. Variances are amenable to mathematical analysis. In particular the additive property of variances with identical dimensions facilitates the calculation of precision estimates for variables and functions.

Standard deviations, confidence intervals and confidence ranges, which have the same dimension as variables and functions, are more transparent measures for precision than

variances. The coefficient of variation is numerically equal to the standard deviation as a percentage of the variable. Its dimension simplifies comparisons of precision estimates for each stage in a measurement chain, the total precision of a measurement procedure, or the precision between different measurement procedures.

Statistical techniques are applied to divide uncertainties into random variations and systematic errors, and to partition the sum of all random variations in a measurement chain into components for various stages. Some systematic errors are intrinsic to sampling, and others to sample preparation. For example, chip samples often display a positive bias (too high a metal grade) due to attrition of mineralised zones inside drill holes while core samples are more likely to display a negative bias (too low a metal grade) due to incomplete core recovery, or abrasion of minerals from the core's surface.

Coarse native metals may lodge on hot grinding surfaces of friction pulverisers until they become dislodged, often at random times but invariably after cooling periods. Oxidation during comminution causes a negative bias while dust loss during comminution and division causes a positive bias. During exploration stages, however, oxidation and loss of dust during sample preparation are not likely to impact significantly on grades and contents of ore deposits.

## PROPERTIES OF VARIANCES

The variance is the basic measure for random variations in a stochastic variable such as a metal's distribution in an ore deposit, in a sample from the deposit, in a test sample from a drill core section, a mass of crushed ore or drill chips, or in a test portion from a test sample. How to obtain at the lowest possible cost reliable variance estimates, for each stage and for the entire chain, and how to optimise the amount of information that exploration programs typically provide, are elements of metrology, the science of measurement, as applied to mining and metallurgy in general, and to exploration and development in particular.

The advantage of the variance as a measure for the intrinsic variability of a metal's distribution, and for the precision of sampling, sample preparation and assaying, is that variances for metal grades and contents are amenable to mathematical analysis. The additive property of variances in particular plays a important role in the calculation of precision estimates for grades and contents of ore deposits.

The variance for a general function of a set of independent variables can be calculated from the variances for all variables in the set. For example, the metal content of a quantity of ore is a simple function of its volume, in-situ density, metal grade and moisture content. In the case that the metal grade is reported in per cent on dry basis the following formula applies:

$$Me = V * D * AF * MF$$

in which :

$Me$	=	metal content in $mt$
$V$	=	volume in $m^3$
$D$	=	in-situ density in $mt/m^3$
$AF$	=	grade factor : %a/100
%a	=	metal grade in per cent on dry basis
$MF$	=	moisture factor: $(100 - \%H_2O)/100$
$\%H_2O$	=	moisture in per cent

Merks and Merks (1991) use the formula for the variance of a general function to calculate the variance for the metal content of

1. President, Matrix Consultants Limited, PO Box 697, Station A, Vancouver, British Columbia, V6C 2N5, Canada.

a volume of in-situ ore. The variance for this function is the sum of squares of partial derivatives with respect to volume, density, grade and moisture multiplied with the variance for each variable. Multiplying the term for volume with:  $V^2/V^2$ , density with:  $D^2/D^2$ , grade factor with:  $AF^2/AF^2$ , and moisture factor with:  $MF^2/MF^2$ , dividing each term by  $Me^2$ , and multiplying the sum of the terms with  $Me^2$ , results in the following formula:

$$var(Me) = Me^2 * \left[ \frac{var(V)/V^2 + var(D)/D^2}{var(AF)/AF^2 + var(MF)/MF^2} \right]$$

in which:  $var(Me)$  = variance for metal content:  $(mt)^2$

$$var(V) = \text{variance for volume: } (m^3)^2$$

$$var(D) = \text{variance for density: } (mt/m^3)^2$$

$$var(AF) = \text{variance for grade factor: dimensionless}$$

$$var(MF) = \text{variance for moisture factor: dimensionless}$$

Similarly, the precious metal content in kg for a volume of in-situ ore is calculated from its volume in  $m^3$ , and precious metal grade in kg/mt on dry basis, density in  $mt/m^3$ , and moisture content in per cent, with the formula:

$$Me = V * D * a * MF$$

in which:  $Me$  = metal content in mt

$$V = \text{volume in } m^3$$

$$D = \text{in-situ density in } mt/m^3$$

$$a = \text{precious metal grade in kg/mt on dry basis}$$

$$MF = \text{moisture factor: } (100 - \%H_2O)/100$$

$$\%H_2O = \text{moisture in per cent}$$

so that the variance for the function between volume, density, grade, moisture and precious metal content in kg becomes:

$$var(Me) = Me^2 * \left[ \frac{var(V)/V^2 + var(a)/a^2 + var(D)/D^2 + var(MF)/MF^2}{var(AF)/AF^2} \right]$$

in which:  $var(Me)$  = variance for precious metal content:  $(kg)^2$

$$var(V) = \text{variance for volume: } (m^3)^2$$

$$var(D) = \text{variance for density: } (mt/m^3)^2$$

$$var(a) = \text{variance for precious metal grade: } (kg/mt)^2$$

$$var(MF) = \text{variance for moisture factor: dimensionless}$$

During exploration stages the effect of moisture on metal content need not be taken into account. For the small negative bias causes the metal content of an ore deposit to be conservative so that the risk to encounter less than the estimated value is marginally reduced. In massive sulphides the correlation between in-situ density and mass should be taken into account but in many gold deposits the variability in density can be ignored. Possible effects of dependencies between densities and either grades or moisture contents should be carefully considered, and taken into account if necessary.

Almost invariably the variance for grade is the dominant component of the variance for content so that the terms:  $var(AF)/AF^2$ , and:  $var(a)/a^2$ , contribute most to the variances for metal contents. Variances for volume and density can be estimated at low cost. How to estimate variances for metal grades at affordable cost, and how to compute variances for metal contents effectively, are the key to reliable precision estimates for ore reserves.

## SAMPLING THEORY

Merks and Merks (1991) define an elementary unit as a quantity of in-situ ore from which a part is extracted, prepared and assayed for metal grade(s). The ratio between the volume or mass that is extracted for a sample, and the volume or mass of in-situ ore, is a

probability to which the term 'sampling ratio' is applied. Sampling ratios in exploration and development generally range from 1:100 000 to 1:1 000 000. Crushed ore from a round in a drift is an elementary unit to which a sampling ratio of 1:1 applies. Evidently, this sampling ratio reflects the high degree of confidence that bulk samples instill. For consignments in bulk the sampling ratio ranges from 1:100 to 1:25 000.

The sampling ratio's reciprocal is the complete set of all possible samples into which each elementary unit can be divided. If an elementary unit is a round from a drift, then the set has only a single element or member. All possible samples in a complete set, when extracted without gain or loss in mass, are unbiased by definition. However, some samples in the set are more precise than others, and thus closer to that elusive unknown true value.

Typical examples of elementary units are a cylindrical volume in an isotropic ore structure, a polygonal volume in an anisotropic ore structure, and a round in a drift that intersects an ore zone. From each elementary unit a section of drill core, a mass of drill chips or a mass of crushed ore, is extracted. If elementary units are rounds in a drift that intersects an ore zone, metal grades can be estimated with the highest degree of precision. For the entire mass of crushed ore from a round, and thus the complete set of all possible drill core sections, or all possible samples of drill chips, is extracted for a sample.

Various techniques have been developed to optimise the amount of information that a large exploration project typically provides. Some techniques create information where none existed which is only acceptable if the probability for continued mineralisation between measured data points is high. However, if this probability is low, then the risk to encounter significantly less than the estimated grade or content during mining becomes high. For example, kriging (Krige, 1976) is most reliable for porphyry type deposits but less effective for gold and massive sulphides deposits.

Geological interpretation and statistical analysis can be applied to quantify the probability for mineralisation to be continuous between measured data points. The question whether 'geostatistics' is an adequate link between geology and applied statistics is intriguing. David (1977) concedes that his book is not for professional statisticians, and that it is written for mining engineers and geologists who usually have little exposure to probability and statistics. Merks (1989) stresses that geostatistical techniques, even if the most powerful computer and the most advanced software were utilised, cannot possibly improve the precision beyond limits that are imposed by the theory of mathematical probability and applied statistics, and that conventional statistical techniques provide powerful tools.

Evidence suggests that geostatistics digressed from applied statistics without recourse to powerful tools and techniques such as analysis of variance and Student's t-test. As a result geostatistical applications of questionable validity have been proposed and published. Applied statistics too is often abused with reckless abandon (Huff, 1954; Reichmann, 1961). Therefore, some knowledge and understanding of applied statistics and geostatistics would assist those involved in mineral exploration and development to recognise commonalities, differences and limitations, and to differentiate between use and abuse.

Matheron (1963) developed his theory of regionalised variables while exploring how spatial correlations between a set of data points in a three-dimensional sample space impact on the variances between aligned or ordered linear subsets. Fisher's F-test can be applied to check whether a spatial correlation is statistically significant. If the calculated F-ratio between the variances for randomised and ordered data sets exceeds a tabulated value, either at 95 per cent or 99 per cent probability and with appropriate degrees of freedom, the variances are incompatible, and a statistically significant spatial correlation exists.

In a perfectly isotropic ore structure the F-ratios between variances for all possible linear subsets of randomised and ordered data that intersect a single point are statistically identical. In an anisotropic ore structure, however, such F-ratios are a function of orientation.

The calculation of variances for metal grades and contents, and the application of the grade-squared partition technique (Merks and Merks, 1991), also require that randomised and ordered linear subsets be checked for spatial correlations. However, only the first term of the space series variances is required for the grade-squared partition technique. This term is first used to calculate the variances for the average metal grade and for cumulative metal content of all elementary units that a set defines. Next, the grade component of the variance for the metal content of the complete set is partitioned into grade components for all elementary units. Finally, each grade component is converted into a variance estimate for the metal grade of each elementary unit in the set.

If the F-ratio between the variances for randomised and ordered sets exceeds the tabulated value at 95 per cent probability, then the first term of the space series variances is significantly lower than the variance for the randomised set. Hence, the spatial correlation results in significantly higher degrees of precision for grade and content. However, if the F-ratio does not exceed this tabulated value, then the variance for the randomised set and the first term of its space series variances are statistically identical. In the case that the first term is only marginally but not significantly lower than the variance for the randomised set, it should be used to calculate the precision for grade and content. However, the conclusion that the spatial correlation results in a significantly higher degree of precision would be invalid.

Fisher's F-test should always be applied before differences between variances are accepted as valid estimates for other variance components. For example, David (1989) enters the difference of 0.162 between variances of 0.45 and 0.288 into his smoothing relationship (David, 1977) which appears to predict a recovery of 52 per cent at 2.15 g/t. However, the variance of 0.45 is based on a set of nine data points, and infinite degrees of freedom were, generously and perhaps ironically, applied to the kriging variance. Nevertheless, the calculated F-ratio of:  $0.45/0.288 = 1.56$  is still below the tabulated value of  $F_{0.95;8; \infty} = 1.94$  which implies that the difference of 0.162 between variances of 0.45 and 0.288 is only a random number. Applying mathematical analysis to such a difference is an exercise in futility.

## SAMPLING PRACTICE

The process of selecting a position and orientation for a drill hole, extracting core sections or drill chips, preparing test samples, and collecting and assaying one or more test portions, generates grade estimates for all elementary units that a drill hole defines. Each stage in this measurement process contributes a variance component, not only to  $\text{var}(\text{AU})$ , the metal's intrinsic variability but also to  $\text{var}(t)$ , the total variance for the metal grade of an elementary unit which reflects this intrinsic variability.

The most important extrinsic components of the total variance are  $\text{var}(s)$ , the sampling variance,  $\text{var}(h)$ , the variance for splitting core sections into halves,  $\text{var}(p)$ , the variance for preparing test samples, and  $\text{var}(a)$ , the variance for collecting and assaying test portions.

Figure 1 shows the selection stage of the measurement process. The drill's position and orientation account for the sampling variance as a measure for random variations at the selection stage. For it defines the core sections or the mass of chips that will be extracted from the drill hole. Theoretically, an estimate for the sampling variance could be obtained by extracting more core sections from the same elementary unit but this technique is

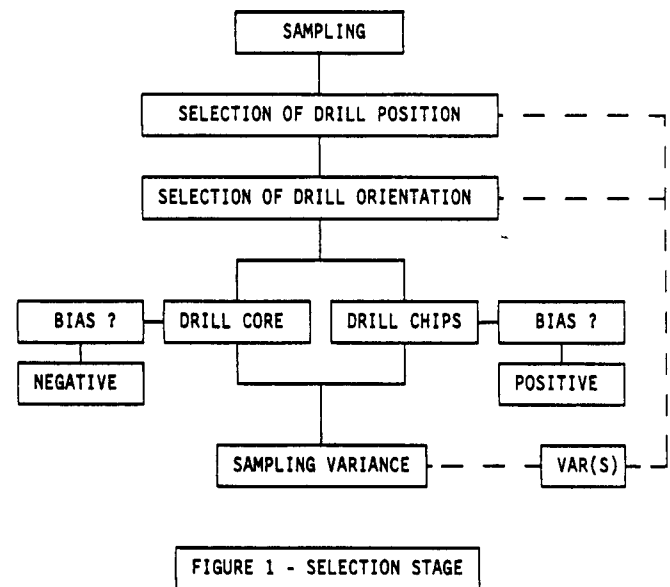


FIG 1

too expensive in practice. The symbol ' $\text{var}(\text{AU})$ ' will be used in reference to the intrinsic variability of a metal's distribution. In mineral exploration the sampling variance is invariably either the largest or the second largest component of the total variance.

Core sections from drill holes are only unbiased if they are perfectly round and quantitatively extracted. Selective abrasion from mineralised sections, and incomplete recovery of drill core from mineralised intersections, cause a negative bias that results in systematically low metal grades and contents. Abrasion is more insidious than incomplete recovery.

Chip samples too are only unbiased if holes are perfectly round and chips are quantitatively recovered. Attrition of mineralised zones results in a positive bias while cavities at mineralised intersections cause a negative bias. Coarse gold that is liberated during drilling may not be completely recovered while molybdenum sulphide migrates in an erratic manner. Porphyry deposits often display a positive bias which results in systematically high metal grades and contents. Figure 1 reflects the most probable potential for positive and negative systematic errors in core and chip samples.

Although metal grades of crushed ore from rounds in drifts can be estimated with a high degree of precision, the process of collecting gross samples from crushed ore is prone to systematic errors. Massive sulphides in particular are friable so that fine fractions invariably contain anomalously high grades. The sampling theory for bulk materials should be applied to identify possible and probable sources for systematic errors when sampling crushed ore.

Figure 2 shows the preparation stage for the measurement process. The variance for splitting core sections into halves is a measure for random variations in this imprecise selection process. The variance of preparation is a measure for random variations in preparing a test sample from a chip sample, a halve core section or a whole core section.

Dividing drill core sections into halves contributes a large extrinsic component to the total variance. During exploration and development core is usually split into halves but during production whole core is often prepared and assayed. The variance for splitting core sections into halves, preparing a test

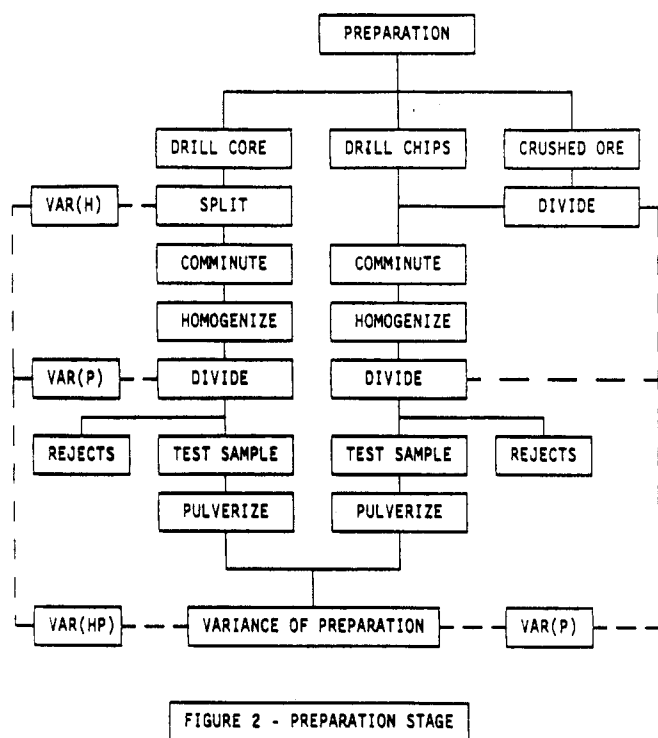


FIGURE 2 - PREPARATION STAGE

FIG 2

sample from each halve, and assaying one or more test portions from a test sample, should be taken into account when calculating variances for metal contents.

Based on a statistical evaluation of a set of gold assays Koch and Link (1970) claim that 'little is gained by assaying the whole core rather than only one-half of it'. Merks (1985) uses this set to show that the variance between core halves is equivalent to a coefficient of variation of 13 per cent against 43 per cent for the variance between core sections within the drill hole, and he estimates that the variance for splitting core sections into halves in terms of a coefficient of variation ranges from a low of ten per cent to a high of 18 per cent.

Even though the F-ratio of:  $4505.7/2279.7 = 1.98$  between the variances for randomised and ordered sets does not exceed a tabulated value of  $F_{0.95;11;23} = 2.24$ , the variance of 2279.7 translates into a coefficient of variation of 29 per cent against 13 per cent for the variance for splitting, preparing and assaying. The variance for splitting, preparing and assaying halve core sections turned out to be equivalent to 20 per cent of the first term of the space series variances. Evidently, the variance for splitting core into halves adds a large variance component to the measurement process so that it makes sense to deduct this variance before precision estimates for metal grades and contents are calculated.

Exploration samples often consist of drill chips that can be divided with a much higher degree of precision than core sections. In other words, for the same ore structure the variance for dividing core sections into halves is significantly higher than the variance for dividing drill chips into equal parts. The symbol 'var(hpa)' applies to the variance for splitting a core section, preparing a test sample from one halve, and collecting and assaying a test portion from a test sample. The symbol 'var(pa)' applies to the variance for dividing a dried, comminuted and homogenised sample (either chips or halve core) into a test

sample and rejects, preparing a test sample, and then collecting and assaying a test portion from a test sample. Each division stage contributes a variance component to the variance of preparation. Comminution stages, by contrast, do not add to the variance of preparation but are more prone to systematic errors than division stages.

The sum of var(AU), the intrinsic variability of a metal's distribution in its matrix expressed as a variance, and var(s), the sampling variance, can be estimated by deducting the variance for splitting, preparing and assaying from the total variance. Similarly, the variance of preparation can be estimated by deducting the variance of assaying from the variance of preparation and assaying. The question is then whether:  $\text{var}(t) - \text{var}(hpa)$ , and:  $\text{var}(hpa) - \text{var}(a)$ , are unbiased estimates for:  $\text{var}(A|U) + \text{var}(s)$ , and  $\text{var}(hp)$ . The F-test should be applied to check whether calculated F-ratios of:  $\text{var}(t)/\text{var}(hpa)$ , and:  $\text{var}(hpa)/\text{var}(a)$ , exceed tabulated F-values, at 95 per cent or 99 per cent probability and with appropriate degrees of freedom. Unless calculated F-ratios exceed tabulated F-values such differences are random numbers to which no significance can be attached let alone mathematical analysis applied.

The variance for splitting, preparing and assaying,  $\text{var}(hpa)$ , can be partitioned into  $\text{var}(hp)$  and  $\text{var}(a)$  by preparing a test sample from each half core section, and assaying duplicate test portions. The mean of absolute differences between average grades of half core sections is a measure for  $\text{var}(hpa)$  while the mean of absolute differences between duplicate assays is a measure for  $\text{var}(a)$ . One could partition  $\text{var}(pa)$  into  $\text{var}(p)$  and  $\text{var}(a)$  by first dividing comminuted and homogenised subsamples into equal parts, and then dividing each part until a test sample is obtained. However, the variance for splitting core significantly higher than the variance for preparing it. Therefore, reliable estimates for  $\text{var}(hpa)$  and  $\text{var}(a)$ , preferably as a function of grade, are more useful to evaluate and optimise the precision for grades and contents. A case can be made that a limited number (2.5 - 5 per cent) of half core sections, with higher than average metal grades, be sacrificed to estimate  $\text{var}(hpa)$  and  $\text{var}(a)$  as a function of grade.

Comminution liberates minerals from their matrix, and results in a higher composition variance. Comminution also increases the number of particles which, for a given mass, results in a lower composition variance. The formula for the binomial distribution shows that the variance for a perfect mixture of two components is a function of the number of particles which implies that the composition variance is mass dependent, and that it can be defined for a mass of 1 kg. If binomial or multinomial populations of particles were perfectly mixed, the distribution heterogeneity, and thus the distribution variance, would be zero by definition (Merks, 1985).

In practical applications the sampling variance almost invariably consist of composition and distribution components. Therefore, comminuted samples should be homogenised prior to division in order to reduce the distribution heterogeneity to a minimum. An effective mixing stage ensures that the mass of a test sample determines its composition variance, and thus the variance for dividing it after mixing.

Visman (1962) explains that the sampling variance consists of composition and distribution components, and that these components can be estimated from sets of small and large increments. Merks (1985) suggests that the variance of analysis for small increments, and the variance of preparation and analysis for large increments be deducted prior to partitioning. Gy's (1979) fundamental error is derived from the variance of the binomial distribution. His sampling error too is a true variance, and not a systematic error or bias. When applying Gy's formula one should carefully consider whether the sample from which his liberation, mineralogical, shape and size factors were estimated was unbiased and precise with acceptable limits for the variable of interest.

During comminution large particles with anomalous distributions disintegrate into clusters of small particles that remain unevenly distributed until the sample is homogenised. Mixing by coning is the most effective procedure to reduce the distribution heterogeneity of comminuted samples, and thus the variances for successive division stages. Unless a comminuted sample is properly mixed the variance for dividing it with a Jones riffle may well be of the same order of magnitude as the variance for splitting core sections. Mixing by coning also combines logically with incremental division or increment reduction, a division procedure that is more precise and less prone to systematic errors than Jones riffles and face dividers.

Figure 3 shows the assay stage of the measurement process. Unless a test sample is perfectly homogeneous the process of collecting a test portion will add an additional component to the variance for assaying. Collecting a number of small increments for a test portion would reduce this variance component. Mixing a test sample reduces its distribution heterogeneity so that the variance for collecting a test portion becomes an insignificant part of the variance of assaying. Test samples may segregate after preparation so that mixing before test portions are collected in the laboratory could result in marginally more precise assays.

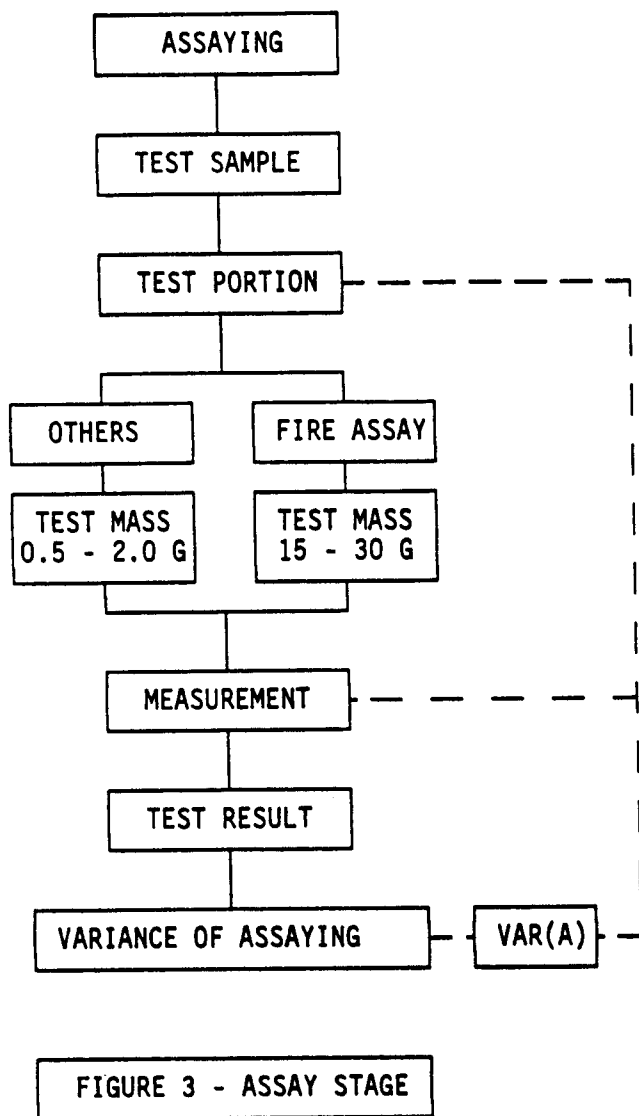


Fig 3

Test samples that contain native metals often require screening at 100 - 150 mesh and separate assays of coarse and fine fractions to obtain an acceptable degree of precision. This method improved the precision of assaying for gold ore with a grade of 0.5 - 1 oz/st from a coefficient of variation of 50 per cent to less

than five per cent. In this case, the term 'precision of assaying' is deceptive. After all, the process of collecting test portions largely accounts for the low degree of precision.

The practice of cutting gold grades that exceed certain limits to some fixed grade is deceptive, discriminatory and disturbing, and should be replaced with replication and statistical rejection criteria. The first step is to collect and assay two more test portions from the same test sample that generated an anomalously high gold assay. The second step is to collect and prepare another test sample, either from the rejects or from the other half of the core, and to collect and assay duplicate test portions. The third step is to apply Dixon's rejection criterion for outliers (Volk, 1980) to check whether the set of assays is homogeneous. If the set is homogeneous, the average gold grade is accepted. If the set fails Dixon's test, the highest assay is rejected, and the average for the reduced set is accepted.

In the case that the set fails Dixon's test a suitable alternative would be to combine and screen at 100-150 mesh the remainders of both test samples, weigh coarse and fine fractions, assay each fraction separately, and to accept the weighted average grade. Even though this alternative is expensive it provides precise and reliable estimates that could make a difference to the viability of marginal deposits.

## SUMMARY

The objective of exploration projects is to estimate metal grades and contents of ore deposits in an unbiased manner, and with a degree of precision that reduces to an acceptable level the risk to encounter less than predicted grades and contents during mining. Yet experiences in recent years have shown that this risk remains high so that not all exploration and development projects meet these criteria.

Meaningful precision estimates for the cumulative metal content of an ore body can be calculated from the sum of the variances for the metal contents of all elementary units. Confidence intervals and ranges at different probability levels are powerful measures for the risk that an ore deposit will contain less than the estimated grade or content. For example, a deposit with an estimated gold content of 50 mt, for which a 95 per cent confidence interval (95 per cent CI) of  $\pm 5$  mt, or  $\pm 10$  per cent, and a 95 per cent confidence range (95 per cent CR) from 45 mt to 55 mt were calculated, would present an acceptable risk. For the same deposit a 95 per cent CI of  $\pm 25$  mt, or  $\pm 50$  per cent, and a 95 per cent CR from 25 mt to 75 mt, would present an unacceptable risk. Asymmetrical confidence ranges in particular are an effective measure for the risk to encounter less than the predicted metal grade or content during production.

The various stages of an exploration project that is designed to estimate the precision of metal grades and contents by applying the grade-squared partition technique are:

- implement a quality assurance program to verify that metal grades are determined in an unbiased manner (all test samples to be assayed by a competent assay laboratory, and five to ten per cent of all test samples, preferably with above average grades, to another reputable assay laboratory), and to estimate the variance for preparation and assaying;
- check test results for outliers, and replace rogue measurements with more reliable estimates by applying statistical rejection criteria;
- check linear subsets (for intersections of drill holes with large polygons or cylinders) for the existence of a serial

correlation by testing for statistical significance the F-ratios between variances for randomised and ordered data;

- deduct the variance for (splitting), preparing and assaying from the lowest variance for each large cylinder or polygon;
- calculate the volume and mass of the large cylinder or polygon that each subset defines;
- calculate the variance for the metal content of the set of elementary units (small polygons or discs) that each large polygon or cylinder defines from its volume, (average) in-situ density, average metal grade, (average moisture content), and the variance for each variable;
- calculate the grade component of the variance for the metal content of each set of elementary units (large polygons or cylinders);
- partition the grade component for the set into grade components for all elementary units that a large cylinder or polygon defines by applying the grade-squared partition technique;
- convert the grade component of each elementary unit into a variance estimate for its metal grade;
- calculate metal contents and variances for metal contents of all elementary units in the set that constitutes the ore deposit, or for all elementary units in the subset that constitutes a logical part of it;
- summate metal contents and variances, either for the entire ore deposit or for a logical part thereof; and
- report precision estimates for metal grades and contents in terms of confidence intervals and ranges.

Table 1 shows how volumes, densities, masses, gold grades and contents for K sets, each consisting of one or more subsets that intersect an ore zone either from surface or from a position in a drift, and their variances, 95 per cent confidence intervals and 95 per cent confidence ranges, can be tabulated.

All measurements for a set are stored in a single spreadsheet file. Variances for randomised subsets are weighted on the basis of length and density while variances for ordered subsets are weighted on the basis of length only. Test data for all sets are combined into a master file. If required simulations can be applied to single files and condensed into a master file.

The sum of variances for gold contents of all sets, which reflects the effect of the Central Limit Theorem, can then be converted into precision estimates for the cumulative gold content in terms of confidence intervals, and symmetrical or asymmetrical confidence ranges. In Table 2 are listed the cumulative gold content, the sum of the variances for the gold contents of all sets, the 95 per cent confidence interval, and the symmetrical 95 per cent confidence range.

Sampling and applied statistics play a key role in metrology, the science of measurement, as it applies to mining and metallurgy. Metrology deals with testing measurement systems and procedures for bias, with estimating the variance for each stage in a measurement chain, and with investigating the interactions between variables and variances. When applied to exploration and development it deals with calculating variances for variables such as volumes, in-situ densities, metal grades and moisture contents, and for functions such as metal contents. It also deals with simulation models for metal contents of ore deposits.

TABLE 1

*Precision estimates for gold contents of sets.*

SET	VOLUME	DENSITY	MASS	GRADE	CONTENT	VARIANCE	95% CI	LOW	HIGH
1	11,000	3.10	36,300	0.35	12,705	2,522,000	± 3,175	9,530	15,880
2	8,500	3.00	25,500	0.27	6,855	474,000	± 1,375	5,510	8,260
3	12,400	2.95	36,600	0.22	8,050	933,000	± 1,905	6,120	9,980
...									
...									
K-2	10,500	2.95	31,000	0.39	12,090	1,447,000	± 2,405	9,685	14,495
K-1	9,700	3.01	29,200	0.43	12,555	2,270,000	± 3,105	9,540	15,570
K	8,900	2.98	26,500	0.37	9,815	925,000	± 1,925	7,890	11,740

TABLE 2

*Precision estimates for cumulative gold content.*

PARAMETER	SYMBOL	VALUE
CUMULATIVE GOLD CONTENT in oz	Au	258,930
SUM OF VARIANCES FOR CONTENT in (oz) <sup>2</sup>	var(Au)	37,198,900
STANDARD DEVIATION in oz	sd(Au)	6,100
95% CONFIDENCE INTERVAL	95% CI	± 12,200
95% CONFIDENCE RANGE : LOW	Au - 95% CI	247,000
HIGH	Au + 95% CI	271,000

## REFERENCES

- David, M, 1977. *Geostatistical Ore Reserve Estimation*, (Elsevier Scientific Publishing Company: Amsterdam).
- David, M, 1989. Grade Control Problems Dilution and Geostatistics: Choosing the Required Quality and Number of Samples for Grade Control, *CIM Bulletin*, November.
- Gy, P M, 1979. *Sampling of Particulate Matter, Theory and Practice*, (Elsevier Scientific Publishing Company: Amsterdam).
- Huff, D, 1954. *How to Lie with Statistics*, (Penguin Books: New York).
- Koch, G S and Link, R F, 1970. *Statistical Analysis of Geological Data*, (John Wiley & Sons: New York).
- Krige, D G, 1976. A Review of Development of Geostatistics in South Africa, *Advanced Geostatistics in the Mining Industry*, pp 279-294, (Reidel: Dordrecht).
- Matheron, G, 1963. Principles of Geostatistics, *Econ Geol*, 58, pp 1246-1266.
- Merks, J W, 1985. *Sampling and Weighing of Bulk Solids*, (Trans Tech Publications: Clausthal-Zellerfeld).
- Merks, J W, 1989. Estimating Reserves, *The Northern Miner Magazine*, September.
- Merks, J W and Merks, E A T, 1991. Precision Estimates for Ore Reserves, *Erzmetall* 44, Nr 10, pp 498 - 505.
- Reichmann, W J, 1961. *Use and Abuse of Statistics*, (Penguin Books: New York).
- Visman, J, 1962. Towards a Common Basis for the Sampling of Materials, Canadian Centre for Mineral and Energy Technology, CANMET Research Report R93, July.
- Volk, W, 1980. *Applied Statistics for Engineers*, (Rober R Krieger Publishing Company: Huntingdon).