Precision estimates for ore reserves

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A methodology to compute precision estimates for ore reserves is introduced. From the measurement of an ore body's volume, metal grade, in situ density, moisture content, and the variances of those measurements, an estimate for the composite variance of total metal content is computed and reported in terms of a confidence interval or range. The variance contributions of the metal grades to the composite variance of total metal content, is partitioned using a grade-squared partition technique, providing variance estimates for the metal grades of the individual quantities of ore that together constitute an ore body. The variance of sampling, preparation and analysis is taken into account when computing variance estimates. The advantage of the grade-squared approach is that metal contents and their variances are additive with respect to summation over individual ore quantities. This simplifies simulation of precision estimates, sensitivity analysis, optimization of mine models, and ultimately, the analysis of the risk of progressing from prospect to producing mine.

● **Genaue Abschätzung von Erzvorräten.** Eine Methodik für die genaue Berechnung bei der Abschätzung von Erzvorräten wird vorgestellt. Aus den Angaben des Erzkörpervolumens, dem Metallgehalt, der insitu-Dichte, dem Feuchtigkeitsgehalt und den Schwankungen dieser Messungen wird ein Schätzwert der gesamten Varianz des gesamten Metallinhalts errechnet und als sicherer Bereich vorgelegt. Die Varianzbestandteile der Metallgehalte im Verhältnis zur Varianz des Gesamtmetallinhalts bedient sich besonders einer saldierten Teilungstechnik, die zu Varianzabschätzungen für die Metallgüten der einzelnen Mengen führt, aus denen ein Erzkörper besteht. Die Schwankungsbreite der Proben, Aufbereitung und Analyse werden bei der Errechnung der Varianzabschätzungen berücksichtigt. Der Vorteil des Gehalts-saldierten Vorgehens liegt darin, daß Metallinhalte und ihre Varianzen additiv sind bezüglich der Summierung einzelner Erzmengen. Dies vereinfacht die Simulation genauer Abschätzungen von Sensitivitätsanalysen, die Optimierung von Grubenmodellen und letztlich die Analyse des Risikos eines Überganges vom Prospektieren zu einer produzierenden Grube.

● **La détermination précise des réserves de minerai**

● **La estimación con precisión de reservas minerales**

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Progression from prospect to producing mine is fraught with scores of uncertainties, not the least of which is due to the measurement process applied to estimate the in situ metal content of a deposit. A mine risks failure if the actual grade of a deposit is significantly lower than the grade that the measurement process implied. The objective of ore reserve estimation techniques [2; 3] is to quantify the volume and mass of an ore deposit and its metal grades. Reliable precision estimates for metal contents, of an entire deposit or large parts thereof, are imperative if the risk of putting an inviable prospect into production is to be reduced to an acceptable level.

Applied statistics provide tools and techniques [8] to compute the variance of a function, such as the metal content of a quantity of ore or concentrate [5]. An advantage of metal contents and their variances is that they are additive with respect to the summation over the quantities of ore that constitute an ore body. The grade-squared partition method uses this property to generate realistic estimates for the variance of the metal grade and metal content of each quantity of ore in a deposit.

The additive property of variances [5; 7] can also be used to partition the variance of sampling, preparation and analysis into its components for each type of sample collected from an ore deposit, at a given stage of exploration or development. Often the correlation between metal grades and the precision of sampling, preparation and analysis is statistically significant. In such cases the variance of sampling, preparation and analysis for the mean grade may be deducted from the variance of the mean grade, either for the complete data set or some subset thereof, to generate an estimate for the measurement-independent (intrinsic) variability of a deposit's metal distribution.

The regression parameters for the correlation between grades and precision can be used to compute a variance estimate for the process of collecting, preparing and assaying different types of samples. Adding this variance to the intrinsic variability of the metal distribution provides an estimate for the variance of metal grade which can then be used to compute an estimate for the composite variance of metal content.

Minerals in an ore deposit often display spatial correlations. Time series analysis [1], and the theory of regionalized variables [3], can both be applied to check whether a set of metal grades in an ore deposit is completely randomized, or whether a spatial correlation exists. The graph that displays space series variances against their spacing is a semi-variogram or sampling variogram [3; 4].

The composite variance for the metal content of an ore deposit is computed either from the variance for the randomized set of metal grades, or from the first term of the space series variances if a signifiant spatial correlation were to exist. Higher terms of the space series variances are not used in this precision estimation technique.

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is deducted from σ_A^2 the variance of the metal grade. Hence, exploration programs should be designed to generate an unbiased estimate for $\sigma^2_{A|spa}$. In the case of drill cores, such an estimate can be obtained if the second half of a limited number of drill core sections displaying higher than average grades in their first half, are prepared and assayed in duplicate. For bulk samples of crushed muck, $\sigma^2_{A|spa}$ can be estimated from the mean of absolute differences between pairs of interpenetrating samples.

With the grade-squared method we will partition var(A | U), the intrinsic variability of the metal distribution, among the elementary units. We will use:

$$\mathrm{var}(\bar{A} \mid U) = \frac{\mathrm{var}_1(A) - \mathrm{var}(A \mid spa)}{n} \tag{14}$$

to estimate the intrinsic variability of the mean grade. Since precision is a function of grade, the regression parameters for the correlation between metal grades and absolute differences can be used to estimate $\sigma^2_{A|spa}$ for each elementary unit. Adding this variance to the intrinsic variability for the metal distribution of each elementary unit provides a realistic estimate for the variance of metal grade which can then be used to compute a composite variance for its metal content.

We will continue our numerical example to show how the grade-squared partition technique produces variance estimates. A significant correlation between metal grades and absolute differences resulted in a regression line with a slope of $m = 0.092$, and an intercept of $b = 0.621$. The variance for a single measurement is equal to:

$$\frac{\pi}{4} \mid \Delta\bar{x} \mid^2. \tag{15}$$

in which $|\Delta\bar{x}|$ is the mean of absolute differences between simultaneous duplicates [5; 8]. Therefore the equation:

$$\mathrm{var}(A \mid spa) = \frac{\pi}{4}(mA + b)^2. \tag{16}$$

provides a variance estimate for $\sigma^2_{A|spa}$.

For a hypothetical elementary unit with the same grade as the average grade of 7.56 g/t we have

$$\mathrm{var}(A \mid spa) = \frac{\pi}{4}(0.092 \cdot 7.56 + 0.621)^2 = 1.36.$$

Deducting this variance estimate, which is significantly lower than the first term of the space series variances, from this term results in

$$\mathrm{var}(A \mid U) = \mathrm{var}_1(A) - \mathrm{var}(A \mid spa) = 10.07 - 1.36 = 8.71$$

so that $\mathrm{var}(\bar{A} \mid U) = 8.71/12 = 0.7258$. Hence,

$$\mathrm{var}(\Sigma Au \mid \bar{A}) = (360 \cdot 3.5 \cdot 0.99)^2 \cdot 0.7258 = 1\,129\,350.$$

The grade-squared partition method partitions the grade component of the composite variance of total metal content to provide estimates for the grade component of each elementary unit in S using the formula:

$$\mathrm{var}(Me \mid A) = \frac{A^2}{\sum_{A \in S} A^2} \mathrm{var}(\Sigma Me \mid \bar{A}). \tag{17}$$

The grade component is used to estimate $\sigma^2_{A|U}$ according to the Formula:

$$\mathrm{var}(A \mid U) = \frac{\mathrm{var}(Me \mid A)}{(V \cdot D \cdot MF)^2} \tag{18}$$

Applying these Formulas to Round 1 gives:

$$\mathrm{var}(Au \mid A) = (2.6569/1\,050.5324) \cdot 1\,129\,349 = 2856$$
$$\mathrm{var}(A \mid U) = 2856/(30 \cdot 3.5 \cdot 0.99)^2 = 0.2643$$
$$\mathrm{var}(A \mid spa) = \pi/4(1.63 \cdot 0.092 + 0.621)^2 = 0.4668$$
$$\mathrm{var}(A) = \mathrm{var}(A \mid U) + \mathrm{var}(A \mid spa) = 0.2643 + 0.4668 = 0.7311$$

Table 1 summarizes the gold grades and the variance terms for each round in our numerical example. Hence, the variance for the measurement process is added to the intrinsic variability of the metal grade to obtain an estimate for the variance of the metal grade.

Whereas a metal grade is a clear and concise measure, its precision in terms of a variance is much less transparent. Converting the variance into a more transparent precision parameter, such as a coefficient of variation, a confidence interval, or a confidence range, puts into perspective the true variability of gold grades. For each round, Table 2 lists the metal grade in g/t, the variance in g^2/t^2, the standard deviation in g/t, the coefficient of variation in %, and the 95% confidence intervals (CI's) and ranges (CR's) in g/t. Coefficients of variation, confidence intervals, and confidence ranges, are well-suited for reporting the precision of metal grades and contents whereas variances are amenable to mathematical analysis.

Table 2
Precision parameters for gold grades in rounds

Round	A	var(A)	sd(A)	CV(A)	95%CI	95%CR low	95%CR high
1	1.63	0.7311	0.8551	52.4	± 1.71	0	3.3
2	3.55	1.9590	1.3997	39.4	± 2.80	0.8	6.3
3	2.72	1.3322	1.1542	42.4	± 2.31	0.4	5.0
4	2.11	0.9647	0.9822	46.5	± 1.96	0.1	4.1
5	4.67	3.0367	1.7426	37.3	± 3.49	1.2	8.2
6	10.78	13.6041	3.6884	34.2	± 7.38	3.4	18.2
7	13.35	20.4167	2.5185	33.9	± 9.04	4.3	22.4
8	13.85	21.9050	4.6803	33.8	± 9.36	4.5	23.2
9	19.69	43.2183	6.5741	33.4	±13.15	6.5	32.8
10	9.32	10.3584	3.2184	34.5	± 6.44	2.9	15.8
11	4.39	2.7523	1.6560	37.7	± 3.31	1.1	7.7
12	4.64	3.0043	1.7333	37.3	± 3.47	1.2	8.1

CV = %. CI = g/t. CR = g/t

Formula (6) can be used to determine the variance components that each measurement contributes to var(Au), the composite variance of metal content for each elementary unit in S. Table 3 summarizes grade, volume, density and moisture factor components together with the composite variance for gold, as it applies to our numerical example.

The grade-squared partition method associates with each elementary unit in an ore body, a metal content and a composite variance for this metal content, togeto estimate the composite variance for total metal content. In the case that a spatial correlation does not exist the *CLT* should be applied to var(A) as follows:

$$\mathrm{var}(\bar{A}) = \frac{\mathrm{var}(A)}{n} \tag{10}$$

to estimate this composite variance.
We will first use $\mathrm{var}(\bar{A}) = \mathrm{var}_1(A)/n = 10.07/12 = 0.8392$ for the ordered set, and later $\mathrm{var}(\bar{A}) = \mathrm{var}(A)/n = 33.18/12 = 2.7650$ for the randomized set, to compute precision estimates for the total gold content of all 12 rounds. Suppose that the volume of each elementary unit is 30 m^3, in situ density is 3.5 t/m^3, and moisture content is 1%. Suppose too that the precision of each measurement, in terms of a coefficient of variation, is 2.5% for volume, 5% for in situ density and 10% for moisture. Thus for each elementary unit we have:

$$\mathrm{var}(V) = (30\cdot 2.5/100)^2 = 0.5625$$
$$\mathrm{var}(D) = (3.5\cdot 5/100)^2 = 0.0306$$
$$\mathrm{var}(MF) = (1\cdot 10/100)^2 = 0.0100$$

Applying Formula (6) to this numerical example gives:

$$\mathrm{var}(\Sigma Au \mid \Sigma V) = (7.56\cdot 3.5\cdot 0.99)^2\cdot 12\cdot 0.5625 = 4632$$
$$\mathrm{var}(\Sigma Au \mid \bar{A}) = (360\cdot 3.5\cdot 0.99)^2\cdot 0.8392 = 1305801$$
$$\mathrm{var}(\Sigma Au \mid \bar{D}) = (7.56\cdot 360\cdot 0.99)^2\cdot 0.0306/12 = 18512$$
$$\mathrm{var}(\Sigma Au \mid \overline{MF}) = (7.56\cdot 360\cdot 3.5)^2\cdot (0.0100/12)\cdot 10^{-4} = 8$$
$$\mathrm{var}(\Sigma Au) = 1328953$$

The gold content of S with a total volume of $12\cdot 30 = 360$ m^3 is

$$7.56\cdot 360\cdot 3.5\cdot 0.99 = 9430 \text{ g.}$$

The composite variance of $\mathrm{var}(\Sigma Au) = 1328953$ g^2 results in a standard deviation of

$$sd(\Sigma Au) = \sqrt{\mathrm{var}(\Sigma Au)} = \sqrt{1328953} = 1153 \text{ g}$$

a 95% confidence interval of $z_{0.95}\cdot \mathrm{sd}(\Sigma Au) \approx 2\cdot 1153 = \pm 2306$ g, or equivalently $(2306/9430)\cdot 100\% = \pm 24\%$; and a 95% confidence range from $9430-2306 = 7.1$ kg up to $9430+2306 = 11.7$ kg. Hence the probability is less than 5% that the gold content of this set of 12 rounds will either be below 7.1 kg or exceed 11.7 kg.
The given precision estimate is an example of a two-sided symmetric 95% confidence range. In the case of ore reserves the one-sided asymmetric lower limit of a confidence range, perhaps at different probability levels, presents a more conservative approach. For example, the lower limits on ΣAu at 10% and 5% probability are

$$\Sigma Au = z_{0.80}\cdot sd(\Sigma Au) = 9430 - 1.282\cdot 1153 = 8.0 \text{ kg}$$
$$\Sigma Au = z_{0.90}\cdot sd(\Sigma Au) = 9430 - 1.645\cdot 1153 = 7.5 \text{ kg.}$$

Hence, the probability is less than 10% that the expected gold content of these rounds will be below 8.0 kg, and less than 5% that it will be below 7.5 kg.
The composite variance of total metal content can also be used to measure the risk of obtaining less than a certain amount of metal. For example, the risk that the set of 12 rounds will contain less than 6.5 kg of gold is measured by computing the z-value of $(9430-6500)/1153 = 2.54$ so that $z = 0.9945$. Hence the risk is less than 0.6% that the expected gold content of 9.4 kg will in fact be less than 6.5 kg.
So far we have been using $\mathrm{var}_1(A)$, the first term of the space series variances to estimate $\sigma^2_{\Sigma Au}$. Without a spatial correlation we can only use var(A), the variance of the completely randomized set, to estimate $\sigma^2_{\Sigma Au}$ which gives us a two-sided 95% confidence interval of ± 4160 g. In that case, the probability would be 95% that the gold content of S is between 5.3 and 13.6 kg. However, for the closely spaced rounds in this decline, the existence of a spatial correlation results in a significantly higher degree of precision, and thus in a lower risk of encountering less than a predetermined amount of gold. For widely spaced drill holes in the same deposit, a spatial correlation between gold grades is not expected to occur. Collecting, preparing and assaying bulk samples from crushed muck is the most precise method to measure the grade for a part of a gold deposit.
In this example, metal grades account for $(1305801/1328953)\cdot 100\% = 98\%$ of the composite variance, volumes add $(4632/1328953)\cdot 100\% = 0.3\%$, in situ densities add $(18.512/1328953)\cdot 100\% = 1.4\%$, and moisture contents add $(8/1328953)\cdot 100\% \approx 0\%$. If the moisture factor were not taken into account when calculating the gold content of these rounds, the resulting systematic error of 0.1 kg would be well within the precision for the chain of measurements, and thus within the precision for contained gold.

3 Precision for metal content of elementary units

The composite variance for the total metal content of a deposit S is equal to the sum of the composite variances of the metal content of each elementary unit in S, i.e.,

$$\sigma^2_{\Sigma Me} = \sum_S \sigma^2_{Me} \tag{11}$$

Substituting variance estimates for the unknown quantities we get:

$$\mathrm{var}(\Sigma Me) = \sum_S \mathrm{var}(Me) \tag{12}$$

The estimated composite variance is also equal to the sum of the variance contributions of the total volume, average grade, average in situ density and average moisture factor, i.e.,

$$\mathrm{var}(\Sigma Me) = \mathrm{var}(\Sigma Me \mid \Sigma V) + \mathrm{var}(\Sigma Me \mid \bar{A}) + \mathrm{var}(\Sigma Me \mid \bar{D}) + \mathrm{var}(\Sigma Me \mid \overline{MF}) \tag{13}$$

An unbiased estimate for the metal grade of each elementary unit is known by definition, and a realistic estimate for its variance can be obtained by applying the grade-squared partition method. The variance of the metal grade of each elementary unit in a set, and thus for the composite variance of its metal content, is estimated by partitioning the grade component $\mathrm{var}(\Sigma Me \mid \bar{A})$ of the composite variance var(ΣMe) into a variance for the metal grade of each unit.
More realistic variance estimates are obtained if the effect of the variance of the measurement process $\sigma^2_{A\mid spa}$

$$\Sigma Me = \Sigma V \cdot \mu_A \cdot \mu_D \cdot \mu_{MF} \tag{5}$$

where ΣV is the total volume, μ_A the mean grade, μ_D the mean density, and μ_{MF} the mean moisture factor. These variables are given in the same units as in Formula (3). Therefore, an estimate for the composite variance of total metal content is given by applying Formula (4) with the appropriate substitution of estimated values for variables to obtain:

$$\mathrm{var}(\Sigma ME) = (\bar{A}\cdot\bar{D}\cdot\overline{MF})^2\mathrm{var}(\Sigma V) + (\Sigma V\cdot\bar{D}\cdot\overline{MF})^2\mathrm{var}(\bar{A}) + (\Sigma V\cdot\bar{A}\cdot\overline{MF})^2\mathrm{var}(\bar{D}) + (\Sigma V\cdot\bar{A}\cdot\bar{D})^2\mathrm{var}(\overline{MF}) \tag{6}$$

The estimate for the composite variance var(ΣMe) is composed of four terms:

$$\begin{aligned}\mathrm{var}(\Sigma Me\,|\,\Sigma V) &= (\bar{A}\cdot\bar{D}\cdot\overline{MF})^2\mathrm{var}(\Sigma V)\\ \mathrm{var}(\Sigma Me\,|\,\bar{A}) &= (\Sigma V\cdot\bar{D}\cdot\overline{MF})^2\mathrm{var}(\bar{A})\\ \mathrm{var}(\Sigma Me\,|\,\bar{D}) &= (\Sigma V\cdot\bar{A}\cdot\overline{MF})^2\mathrm{var}(\bar{D})\\ \mathrm{var}(\Sigma Me\,|\,\overline{MF}) &= (\Sigma V\cdot\bar{A}\cdot\bar{D})^2\mathrm{var}(\overline{MF})\end{aligned} \tag{7}$$

where each term var($\Sigma Me|x$) is viewed as the variance contribution of x, to the composite variance of total metal content. We will use a numerical example to show the relative magnitude of the variance contributions.

A set S of gold assays in 12 rounds from a decline in a gold deposit (see Column 2 in Table 1), will be used to show how to compute an estimate for the composite variance of total metal content. Each gold assay in Column 2 of Table 1 is the average gold assay for either 9 or 18 test samples. Even though coarse gold particles in ore generally display a *Poisson* distribution, average gold assays in sets of test samples tend towards a *Gaussian* or normal distribution. This premise underlies subsequent precision statements.

Table 1
Variances determined by the grade-squared partition method (dimensions see text)

Round	A	A^2	var(Au|A)	var(A|U)	var(A|spa)	var(A)
1	1.63	2.6569	2856	0.2643	0.4668	0.7311
2	3.55	12.6025	13548	1.2538	0.7052	1.9590
3	2.72	7.3984	7953	0.7360	0.5962	1.3322
4	2.11	4.4521	4786	0.4429	0.5218	0.9647
5	4.67	21.8089	23445	2.1697	0.8670	3.0367
6	10.78	116.2084	124927	11.5613	2.0428	13.6041
7	13.35	178.2225	191594	17.7310	2.6857	20.4167
8	13.85	191.8225	206214	19.0840	2.8210	21.9050
9	19.69	387.6961	416784	38.5711	4.6472	43.2183
10	9.32	86.8624	93379	8.6417	1.7176	10.3584
11	4.39	19.2721	20718	1.9173	0.8250	2.7423
12	4.64	21.5296	23145	2.1419	0.8624	3.0043
Σ		1050.5324	1129349			

The average grade of S is $\bar{A} = 7.56$ g/t. Whereas var(A) $= 33.18$ g^2/t^2 for the randomized set, the estimate for the first term of the space series variances between grades of adjacent rounds in the ordered set is $\mathrm{var}_1(A) = 10.07$ g^2/t^2. The F-ratio of $33.18/10.07 = 3.29$ exceeds the tabulated values of $F_{0.99;11,24} = 3.09$ so the probability is less than 1% that the variances for ordered and randomized data are statistically identical. Hence, the set of gold grades, measured in crushed muck from 12 rounds in a decline, displays a significant spatial correlation.

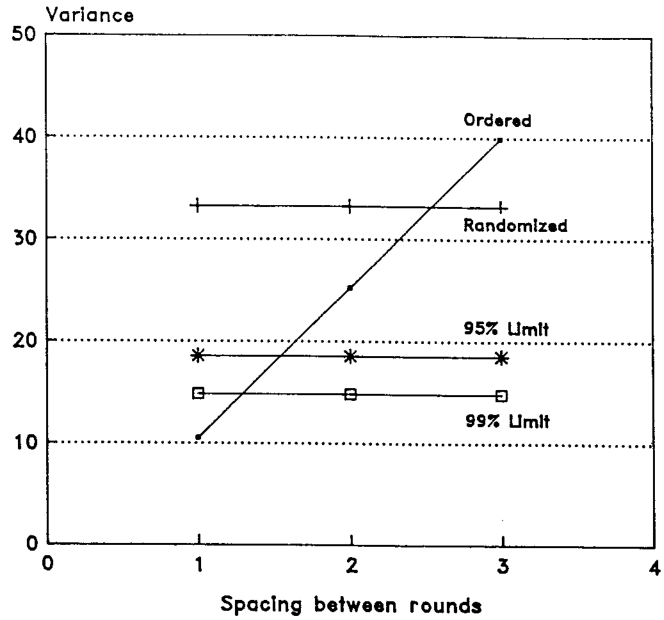

Fig. 1. Variances of gold

In Figure 1 are plotted the first 3 terms of the space series variances, the variance for the randomized set, and the lower limits of its 95 and 99% confidence range. The F-ratio between var(A) and var$_1$(A) is statistically significant, and hence there is evidence of a spatial correlation. However, since the F-ratio between var(A) and var$_2$(A) is not significant, a spatial correlation no longer exists at a spacing of 2 rounds. In fact, based on interpolation in Figure 1, we can state with 99% confidence that a spatial correlation no longer exists at about 6.5 m. Similarly, it is 95% certain that a spatial correlation no longer exists at about 8 m.

One should not attach significance to any terms of the space series variances that are statistically identical to the variance estimate for the randomized set. Only in the case that the first term of the space series variances is below the lower limit of the 95% confidence range for the randomized set, which is equal to $33.18/F_{0.95,11,\infty} = 33.18/1.79 = 18.54$, and preferably below the lower limit of the 99% confidence range which is equal to $33.18/F_{0.99,11,\infty} = 33.18/2.24 = 14.81$, will the spatial correlation result in a significantly more precise estimate for the total metal content of an ore deposit.

Precision estimates for total metal content using the grade-squared partition technique require that the F-ratio between the variance of the randomized set and the first term of the space series variances be checked for statistical significance, i.e.,

$$F_{1-\alpha,n-1,2n-2} = \frac{\mathrm{var}(A)}{\mathrm{var}_1(A)} \tag{8}$$

Thus the inference that a spatial correlation is significant has a probability α of being wrong. Only if the spatial correlation is significant should the *Central Limit Theorem* [8] be applied to var$_1$(A) as follows:

$$\mathrm{var}(\bar{A}) = \frac{\mathrm{var}_1(A)}{n} \tag{9}$$

Fisher's F-ratio test should be applied to check whether the variance between a set of ordered data points is significantly lower than the variance for the randomized set of metal grades within the same geological structure. Unless the *F*-ratio is statistically significant, the mineralization continuity between data points is uncertain; this renders the process of computing additional data points, where none existed, an exercise in futility which is remarkably akin to gambling.

A simple numerical example for a one-dimensional model (a set of rounds in a decline) will be used to show how an estimate for the composite variance of the total metal content of an ore deposit is computed from the estimated variances of its volume, metal grade, in situ density and moisture content. It will also show how different types of confidence intervals and ranges for metal content are calculated, and how the grade-squared partition method is used to estimate a composite variance for each quantify of ore in a deposit.

1 Elementary units

We shall define an *elementary* unit as any quantity of ore from which an unbiased sample is collected, which is then prepared without introducing bias, and measured by applying a reliable analytical procedure. All random variations in the process of collecting a sample, preparing a test sample, and then collecting and assaying a test portion, accumulate to give the variance of sampling (s), preparation (p) and analysis (a) [5; 7]:

$$\sigma^2_{spa} = \sigma^2_s + \sigma^2_p + \sigma^2_a \quad (1)$$

Hence, the variance of sampling, sample preparation and measurement is equal to the sum of a sampling variance σ^2_s, the variance of preparation σ^2_p, and the variance of analysis σ^2_a. Examples of elementary units are: a stratum of in situ ore represented by a section of drill core, a cell within a blast section represented by the chips from a blasthole, or a block in an adit or trench represented by a crushed bulk sample.

The random variations that are independent of the variations resulting from σ^2_{spa} are given by the intrinsic variability of the metal grade for which the variance σ^2_U is a measure. Thus the variance of an elementary unit's metal grade A, is given by:

$$\sigma^2_A = \sigma^2_{A|U} + \sigma^2_{A|spa} \quad (2)$$

where $\sigma^2_{A|U}$ is the intrinsic variability of the metal distribution and $\sigma^2_{A|spa}$ is the variance of sampling, preparation and analysis of the metal grade.

The sampling ratio, i.e., the ratio between the mass of a sample and the mass of the elementary unit it represents, is an intuitive measure of the degree of confidence a sample's metal grade should instill. For example, a chip sample from a face represents a layer with a thickness that matches the particle size of the coarsest chips - a similar metal grade beyond this layer cannot be taken for granted. By contrast, a bulk sample of crushed muck from an adit or trench instills much more confidence in the metal grade of in situ ore. Sampling ratios for exploration and development range from 1:1000 for a bulk sample to more than 1:1000000 for drill core.

The metal content *Me* in grams of an elementary unit in an ore deposit is given by:

$$Me = V \cdot A \cdot D \cdot MF \quad (3)$$

where *V*, *A*, *D* and *MF* respectively denote the volume in m^3, metal grade in g/t, in situ density in t/m^3, and moisture factor as $(100 - \%H_2O)/100$, of an elementary unit. If the metal grade is reported as a percentage, then *AF*, the grade factor, is used in the above Formula as $AF = A/100$.

The measurement of an elementary unit's volume, grade, in situ density and moisture content, each contribute to the composite variance of its metal content. The composite variance of metal content σ^2_{Me} is given in terms of the variances of the volume σ^2_V, the grade σ^2_A, the density σ^2_D and the moisture factor σ^2_{MF}, according to the Formula [8]:

$$\begin{aligned}\sigma^2_{Me} &= \left(\frac{\partial Me}{\partial V}\right)^2 \sigma^2_V + \left(\frac{\partial Me}{\partial A}\right)^2 \sigma^2_A + \left(\frac{\partial Me}{\partial D}\right)^2 \sigma^2_D + \left(\frac{\partial Me}{\partial MF}\right)^2 \sigma^2_{MF} \\ &= (A \cdot D \cdot MF)^2 \sigma^2_V + (V \cdot D \cdot MF)^2 \sigma^2_A + (V \cdot A \cdot MF)^2 \sigma^2_D \\ &\quad + (V \cdot A \cdot D)^2 \sigma^2_{MF}\end{aligned} \quad (4)$$

This Formula is based on the premise that the variables are statistically independent. One would not expect a covariance to exist between the volume of an ore deposit and its metal grade, in situ density or moisture content. A covariance between moisture content and in situ density would have only a marginal effect on the composite variance. Covariances between metal grades, and either the moisture content or the in situ density, will impact on the composite variance, but the effect can be eliminated by applying this formula to uniform geological structures only. If an ore deposit were to consist of different geological structures, the formula would be applicable to each of those structures.

The composite variance of metal content σ^2_{Me} can be used to provide a precision statement for metal content. Furthermore, since metal contents and their variances are additive with respect to summation over elementary units, it is simple to generate a precision statement for the metal content of any set of elementary units, and hence for the complete ore deposit.

A numerical example will be used to show that the precision of volume, in situ density and moisture content impact only marginally on the precision of metal content. Generally, the precision of the metal grades accounts for more than 95% of the composite variance of the total metal content of an ore deposit. Nevertheless, reliable estimates for the variances of volume, in situ density and moisture content, which can be obtained at an affordable cost, are useful to assess their effect on the precision of metal content. For example, a gold deposit may contain large amounts of clay with a highly variable moisture content. Sensitivity analysis by simulation would show how the variance for the moisture factor impacts on the precision for contained gold.

2 Precision for total metal content

Just as the metal content of an elementary unit is given by Formula (3), the total metal content ΣMe of an ore body is given by:

Table 3
Composite variance in g^2/t^2 for gold content of rounds in g

Round	var($Au \mid A$)	var($Au \mid V$)	var($Au \mid D$)	var($Au \mid MF$)	var(Au)
1	7900	18	72	0.03	7990
2	21168	85	340	0.14	21593
3	14395	50	200	0.08	14645
4	10424	30	120	0.05	10574
5	32813	147	589	0.24	33549
6	147000	785	3137	1.28	150923
7	220615	1204	4811	1.96	226632
8	236697	1295	5177	2.11	243171
9	467000	2618	10465	4.27	480087
10	111929	587	2345	0.96	114862
11	29632	130	520	0.21	30282
12	32463	145	581	0.24	33189
Σ	1332036	7094	28357	12.00	1367499

ther with the conventional parameters such as coordinates, volume, in situ density, etc. Metal contents and composite variances for any set of elementary units are additive. For example, the total mass of 312 t for the rounds 10 to 12 contains 1907 g of gold for an average gold grade of $1907/312 = 6.12$ g/t. The composite variance for this metal content is 178333 g^2/t^2 which gives rise to a symmetric 95% confidence interval of $2\sqrt{178333} = \pm 845$ g, for an expected gold content between $1907 - 845 = 1.1$ kg minimum, and $1907 + 845 = 2.8$ kg maximum. Lower limits for asymmetric confidence ranges are:

$1907 - 1.28 \cdot 422 = 1.4$ kg at 10%
$1907 - 1.645 \cdot 422 = 1.2$ kg at 5%
$1907 - 2.055 \cdot 422 = 1.0$ kg at 1%.

The grade-squared partition method allows precision statements to be made with respect to any subset of elementary units so that the total metal content may be reported with precision statements in terms of confidence intervals and ranges.

4 Conclusion

The variances for the measurement of an ore deposit's volume, average metal grade, in situ density and moisture content, can be used to compute a composite variance for metal content and precision statements for metal content and grade. If dependencies between metal grades and in situ densities or moisture contents were to occur, their effect on this composite variance should be taken into account.
The grade-squared partition technique can be applied to partition the grade component of the composite variance for total metal content into a set of grade components corresponding to the elementary units that together constitute an ore deposit, to convert the grade component of each elementary unit into a variance for its metal grade, and to compute the composite variance for the metal content of each elementary unit. The metal contents for all elementary units, and their composite variances, are additive. Hence, for any logical subset of elementary units a metal content and composite variance can be obtained by addition.
Fisher's F-ratio test is applied to check whether the metal grades in a set of elementary units are completely randomized, or whether they are correlated in space. Unless the first term of the space series variances is significantly lower than the variance for the completely randomized set, a spatial correlation between adjacent elementary units is not expected to exist, and mineralization between data points is not necessarily continuous.
Similarly, the F-ratio between the variance for the first term of the space series variances or for the randomized data set, and the variance of sampling, sample preparation and analysis, should be checked for statistical significance. Only in the case that their F-ratio exceeds the tabulated value, is their difference a meaningful variance component rather than a random number.
The question whether the variance of sampling, sample preparation and analysis should be added to the intrinsic variability of a metal in an elementary unit is debatable. After all, one would not encounter the variance for measuring an elementary unit's metal grade when mining the ore but only its intrinsic variability.
The graded-squared partition technique is presented as an alternative to other estimation techniques. Its is similar in the sense that it is a typical computer application. Once all relevant data for a set of elementary units are stored, effective and powerful simulation models can be applied. In particular, if dependencies between metal grades, in situ densities and moisture contents were to occur, conditional simulations that are based on co-variances between variables, can be applied most effectively.

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