Metrology in Mineral Exploration

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Abstract

The objective of metrology in mineral exploration is to derive unbiased confidence limits for masses of metals contained in reserves and resources. Sets of test results for gold and silver determined in pairs of interleaved bulk samples give unbiased confidence limits for the central value of each metal. The same sets of test results show how to verify spatial dependence by applying Fisher's F-test to the variance of a set and the first variance term of the ordered set. A significant degree of spatial dependence gives a higher degree of precision for the central values of gold and silver. Plotting variance terms of an ordered set against the variance of the set and its lower limits of asymmetric 99% and 95% confidence ranges gives a sampling variogram. Sampling variograms put in plain view where spatial dependence in sampling units or sample spaces dissipates into randomness. The application of statistical methods to test results for sets of ordered core samples within boreholes, and for sets of ordered ore volumes within profiles, plays an important role in defining reserves and resources.

Biography

Jan Merks is an author, consultant, lecturer and internationally recognized authority on sampling theory and practice for static and dynamic stochastic systems, and on the application of statistical methods in data analysis. His interleaved sampling protocol takes into account that the variance of the primary sample selection stage is the sum of the composition variance and the distribution variance. His point of view on the additive properties of variances plays a role in several ISO Standards. Visman's sampling theory inspired and underpins much of his work.

Ed Merks leads the Eclipse Modeling Framework project as well as the top-level Eclipse Modeling project. He is a coauthor of the authoritative book "*EMF: Eclipse Modeling Framework.*" He is an elected member of the Eclipse Foundation Board of and has been recognized by the Eclipse Community Awards as Top Ambassador, Top Committer, and Top Newcomer Evangelist. After completing his Ph.D. at Simon Fraser University he spent 16 years at IBM, achieving the level of Senior Technical Staff Member.

Introduction

The variance is the fundamental measure for variability in sampling units and sample spaces. The properties of variances are of critical importance in sampling theory and practice. Variances are amenable to mathematical analysis (Visman, 1962, Volk, 1980, Merks, 1985, Merks and Merks, 1989). Due to its squared dimension the variance is not an intuitive measure for variability. The standard deviation is the square root of the variance and has the same dimension as the central value of a set of measured values. Pearson's coefficient of variation is the standard deviation as a percentage of a central value. Coefficients of variation make it easy to check and compare at a glance the degree of variability in measurement hierarchies, and in stochastic systems such as ore deposits, mined ore and mill feed.

Ordered sets of independently measured values of stochastic variables in sampling units and sample spaces may display associative dependence. The term 'spatial dependence' is used in this context. A significant degree of spatial dependence gives a higher degree of precision for the central value of a set of independently measured values. The adjective ‘independent’ implies that sampling, sample preparation and analytical stages should not cause a significant degree of associative dependence between sequentially measured values in any subset of the whole set.

Analysis of variance is applied to test for spatial dependence by verifying whether or not the first variance term of an ordered set of measured values is significantly lower than the variance of the set. A significant degree of spatial dependence makes it possible to chart a sampling variogram by plotting higher variance terms of the ordered set against the variance of the set, and against the lower limits of the asymmetric 99% and 95% confidence ranges. The lag of a sampling variogram is the X-value of the point where a variance term of the ordered set intersects the lower limit of the asymmetric 95% confidence range.

Confidence intervals and symmetric and asymmetric confidence ranges for masses of metals contained in contiguous sets of blocks are intuitive and user-friendly measures for precision. Confidence intervals are given in absolute units and in relative percentages whereas lower and upper limits of symmetric or asymmetric confidence ranges are given in absolute units only.

Test results for interleaved bulk samples

A specific example of Fisher’s F-test for spatial dependence is given for gold and silver grades of interleaved bulk samples from the Cerattepe deposit located about 200 km by highway from Cayeli in North-Eastern Turkey. Drill core recovery from this soft surficial gossan was deemed unacceptably low. So, it was decided to take bulk samples from a 1 by 2 m pit at 1 m intervals and select a set of primary increments from every 1 m^3 of crushed ore. Each 1 m^3 volume was hoisted to surface where a set of primary increments was selected and partitioned into a pair of interleaved subsets such that one primary sample consisted of all odd-numbered increments and the other of all even-numbered increments. Interleaved samples take into account the second variance term of the ordered set of primary increments. The advantage of interleaved samples is that each pair gives a single degree of freedom.

Selecting from each 1 m^3 of crushed gossan a pair of interleaved primary samples (A- and B-primary samples in ISO parlance) at the excavation stage gives the most reliable estimate for var(spa), the measurement variance. This measurement variance is the sum of var(s), the variance of the primary sample selection stage, var(p), the variance of the sample preparation stage, and var(a), the variance of the analytical stage. The variance of selecting a test portion of a test sample is an integral part of the analytical variance. Measurement variances are extrinsic to intrinsic variances of metals in ore deposits, and may be subtracted before 95% confidence limits for gold and silver grades of this gossan ore are computed.

Primary A- and B-samples were homogenized at the excavation site. Next, A- and B-subsamples were selected and submitted to the assay laboratory for further sample preparation and analysis. Duplicate test portions were selected from all A- and B-test samples and assayed for gold and silver. Sets of test results for gold and silver are given in Excel templates Cerratepe_gold and Cerrateppe_silver.

Table 1 lists the basic statistics for interleaved samples such as the central values for gold and silver, the measurement variances, the standard deviations and the coefficients of variation.

Table 1 – Basic statistics for gold and silver grades

Statistic	Symbol	Gold	Silver
Central value in gpt	$\bar{x}$	10.86	484.6
Measurement variance in gpt^2	var(spa)	0.689	606.3
Standard deviation in gpt	sd(spa)	0.8301	24.6
Coefficient of variation in %rel	CV_{spa}	7.6	5.1

If the mass of excavated gossan, its moisture content and in-situ density are known, 95% confidence limits for masses of contained gold and silver may be derived (Merks, 1985). The methodology is described in standard methods approved by several ISO Technical Committees. Table 2 gives various precision estimates for gold and silver grades in gram per metric ton.

Table 2 – Confidence limits for central values of gold and silver

Statistic	Symbol	Gold	Silver
Central value in gpt	$\bar{x}$	10.86	484.6
95% Confidence interval in gpt	95% CI	±0.05	±9.9
95% Confidence interval in %rel	95% CI	±0.50	±2.0
95% Confidence range:	95% CR		
Lower limit in gpt	95% CRL	10.80	475.0
Upper limit in gpt	95% CRU	10.91	495.0
Measurement variance in gpt^2	var(spa)	0.689	606.3
Number of 1 m^3 blocks	n	26	
Variance of central value in gpt^2	var($\bar{x}$)	0.0265	23.32
Tabulated t-value	t0.05;26	2.056	

The central values in the above table are arithmetic means. The Central Limit Theorem defines the variance of the arithmetic mean as a function of the variance of the set of measured values and the number of measured values in the set. In formula, var($\bar{x}$) = var(spa) ÷ n. Sets of core samples that vary in length and density commonly occur when exploring massive sulfides. The central value is then the length- and density-weighted average of the set of core samples. The Central Limit Theorem should take into account weighting factors for density and length.

Testing for spatial dependence

Testing for spatial dependence is based on the application of Fisher's F-test to verify whether var(x), the variance of the set, and $var_1(x)$, the first variance term of the ordered set, are statistically identical or differ significantly. If these variances are statistically identical, the ordered set of metal grades is randomly distributed within the sample space under examination. If the first variance term is significantly lower than the variance of the set, the ordered set of grades displays a significant degree of spatial dependence. In the latter case, additional variance terms should be tested to assess whether a sampling variogram may be plotted to determine where spatial dependence in the sample space dissipates into randomness.

The formula for the variance of a set indicates that the order in which squared differences are summated does not impact its numerical value. This formula is $var(x) = \Sigma [\bar{x} - x_i]^2 \div [n-1]$ where $\bar{x}$ is the central value of all pairs of interleaved bulk samples, x_i is the mean of the ith pair, n is the number of 1 m^3 volumes of excavated gossan, and n-1 is the number of degrees of freedom for the set.

The first variance term of the ordered set is $var_1(x) = \Sigma[x_i - x_{i+1}]^2 \div 2[n-1]$ where x_i is the mean of the ith pair of interleaved bulk samples, x_{i+1} is the mean of the (i+1) th pair and n the number of 1 m^3 volumes. The first term of the ordered set gives $df_o = 2 \cdot (n-1)$ degrees of freedom (Merks, 1991). The number of degrees of freedom for this first variance term reflects that all but the first measured value and the last are used twice when Riemann sums for this term are derived. By implication, the last Riemann sum of the second variance term is $[x_{n-2} - x_n]^2$. Hence, each higher variance term has one less datum and two fewer degrees of freedom than the previous term.

If the observed value of $F = var(x) \div var_1(x)$ were to exceed the tabulated value of $F0.05;df;df_o$ at 95% probability, of $F0.01;df;df_o$ at 99% probability, or of $F0.001;df;df_o$ at 99.9% probability, then the ordered set exhibits a significant degree of spatial dependence at the corresponding probability level.

The set of arithmetic means of A- and B-interleaved primary samples, selected from twenty-six (26) 1 m^3 volumes of crushed gossan, gives df = 26–1 = 25 degrees of freedom. The ordered set gives $df_o = 2 \cdot (26–1) = 50$ degrees of freedom. The variance of the set of arithmetic mean gold grades is var(x) = 68.91 gpt^2, and the first variance term of the ordered set is $var_1(x)$ = 10.65 gpt^2. The observed value of F = 68.91÷10.65 = 6.47 exceeds the tabulated value of F0.001;25;50 = 2.79 at 99.9% probability. Hence, the ordered set of gold grades displays a highly significant degree of spatial dependence. As a result, the first variance term does indeed give a significantly higher degree of precision for the arithmetic mean gold grade of $\bar{x}$ = 10.86 gpt for this set of bulk samples than would the variance of the set.

The variance of the set of arithmetic mean silver grades of A- and B-primary samples is var(x) = 58,165 gpt^2, and the first variance term of the in-situ ordered set is $var_1(x)$ = 9,602 gpt^2. The observed value of F = 58,165÷9,602 = 6.06 exceeds F0.001;25;50 = 2.79 at 99.9% probability. Hence, the set of in-situ ordered mean silver grades displays a highly significant degree of spatial dependence. By implication, the first variance term of the in-situ ordered set gives a significantly higher degree of precision for the arithmetic mean silver grade of $\bar{x}$ = 484.6 gpt for interleaved bulk samples than the variance of the set.

Table 3 - Test for spatial dependence (measurement variances included)

Statistic	Symbol	Gold	Silver
Variance of set in gpt^2	var(x)	68.91	58,165
First variance term of ordered set in gpt^2	$var_1(x)$	10.65	9,602
Observed F-value	F	6.47	6.06
Significance		***	***
Tabulated F-value at 0.1% probability	F0.001;df;df_o	2.79	
Degrees of freedom for:			
Set	df	25	
Ordered set	df_o	50	

*** significant at 99.9% probability

Table 3 lists the F-statistics for gold and silver obtained by applying Fisher's F-test to the variances of the sets and the first variance terms of the ordered sets before the measurement variances are subtracted. Subtracting measurement variances of var(spa) = 0.6891 gpt^2 for gold and var(spa) = 606.3 gpt^2 for silver from the variances of the sets and of the ordered sets, and applying Fisher's F-test to the intrinsic variances, gives F = $var_i(x) \div var_{i1}(x)$ = 68.22 ÷ 9.96 = 6.85 for gold and F = 57,588÷8,996 = 6.40 for silver. The observed F-values are but marginally higher than those for uncorrected variances. Table 4 lists F-statistics obtained by applying Fisher's F-test after extraneous measurement variances are subtracted.

Table 4 - Test for spatial dependence (measurement variances subtracted)

Statistic	Symbol	Gold	Silver
Variance of set in gpt^2	var(x)	68.22	57,588
First variance term of ordered set in gpt^2	$var_1(x)$	9.96	8,996
Observed F-value	F	6.85	6.40
Significance		***	***
Tabulated F-value at 0.1% probability	F0.001;df;df_o	2.79	
Degrees of freedom for:			
Set	df	25	
Ordered set	df_o	50	

*** significant at 99.9% probability

Subtracting low measurement variances impacts little on lags of sampling variogram. Dividing whole core into halves does give much higher variances so that it makes sense to correct.

Confidence ranges of variances

The lower limit of the asymmetric 95% confidence range for var(x), the variance of the set, is 95% ACRL = var(x) ÷ F0.05;df;∞ where df = 25 for this set of interleaved bulk samples and df = ∞ for infinite degrees of freedom (Volk, 1980). The tabulated value of F0.05;df;∞ = 1.51 would be obtained from the last row for df = ∞ in the F-distribution at 5% probability by interpolation between columns marked df = 24 and df = 30 (Volk, 1980). Excel's FINV function works just as well and much faster when p = 0.05, df = 25 and df = 1,000,000 are entered.

The lower limit of the asymmetric 99% confidence range for var(x), the variance of the set, is 99% ACRL = var(x) ÷ F0.01;df;∞ where df = 25. The tabulated value of F0.01;df;∞ = 1.77 is obtained with Excel's FINV function. Dividing var(x) = 68.91 gpt^2 for gold by F-values at 95% and 99% probability gives 95% ACRL = 68.91÷1.51 = 45.63 gpt^2 and 99% ACRL = 68.91÷1.77 = 38.93 gpt^2 for gold. Similarly, dividing var(x) = 58,164 gpt^2 for silver by F0.05;25;∞ = 1.51 and F0.01;26;∞ = 1.77 gives the lower limits of asymmetric 95% and 99% confidence ranges so that 95% ACRL = 58,164 ÷ 1.51 = 38,520 gpt^2 and 99% ACRL = 58,164 ÷ 1.77 = 32,861 gpt^2 for silver.

The lower variance terms of the ordered sets of arithmetic mean gold and silver grades, the variances of the sets, and the lower limits of asymmetric 95% and 99% confidence ranges, underpin the sampling variograms for gold and silver in this gossan deposit.

Sampling variograms

Fisher's F-test proves that the first variance terms of in-situ ordered mean gold and silver grades of the set of 26 interleaved bulk samples display a significant degree of spatial dependence at 99% probability. Plotting additional variance terms of in-situ ordered arithmetic mean gold and silver grades against the variances of the sets and the lower limits of asymmetric 99% and 95% confidence ranges gives sampling variograms that show where orderliness in this sample space dissipates into randomness.

The formula for j_{th} variance term of the ordered set of mean gold grades of interleaved bulk samples is $var_j(x) = \Sigma [x_i - x_{i+j}]^2 \div 2 \cdot [n-j]$, which gives $var_1(x)$ = 10.65 gpt^2, $var_2(x)$ = 20.22 gpt^2, $var_3(x)$ = 23.12 gpt^2, $var_4(x)$ = 38.14 gpt^2 and $var_5(x)$ = 53.40 gpt^2. The variance terms of the ordered set, the variance of the set, and the lower limits of its asymmetric 95% and 99% confidence ranges, define the sampling variogram and its lag for gold.

Only the first variance term in Table 5 derives from the complete set of test results for interleaved bulk samples. Each next term has one less datum and two fewer degrees of freedom than the previous term.

Table 5 – Variances and lower limits of asymmetric confidence ranges for gold

Statistic	Symbol	$var_j(x)$	var(x)	95% ACRL	99% ACRL
First term	$var_1(x)$	10.65	68.91	45.75	38.87
Second term	$var_2(x)$	20.22	70.82	46.67	39.54
Third term	$var_3(x)$	23.12	73.28	47.92	40.48
Fourth term	$var_4(x)$	38.14	65.58	42.53	35.81
Fifth term	$var_5(x)$	53.40	67.21	43.20	36.25

The first variance term of the ordered set, four variances of the reduced sets, the variance of the set, and matching lower limits of its asymmetric 95% and 99% confidence ranges in Table 5 define the sampling variogram for gold in Figure A.

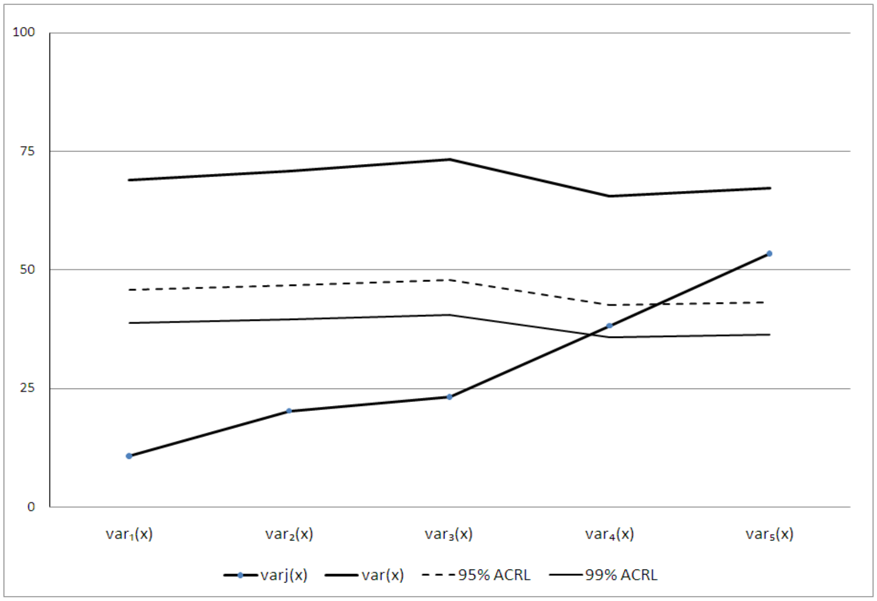

Figure A – Sampling variogram for gold

The same formula gives $var_1(x)$ = 9,602 gpt^2, $var_2(x)$ = 19,716 gpt^2, $var_3(x)$ = 25,187 gpt^2, $var_4(x)$ = 39,924 gpt^2 and $var_5(x)$ = 43,155 gpt^2 for silver. Table 6 lists the variance terms of the ordered sets, the variance of the set, four variances of reduced sets, and matching lower limits of asymmetric confidence ranges at 95% and 99% probability.

Table 6 – Variances and lower limits of asymmetric confidence ranges for silver

Statistic	Symbol	$var_j(x)$	var(x)	95% ACRL	99% ACRL
First term	$var_1(x)$	9,602	58,164	38,619	32,813
Second term	$var_2(x)$	19,716	59,089	38,944	32,955
Third term	$var_3(x)$	25,187	58,763	38,426	32,459
Fourth term	$var_4(x)$	35,924	56,088	36,373	30,626
Fifth term	$var_5(x)$	43,155	57,513	36,968	31,022

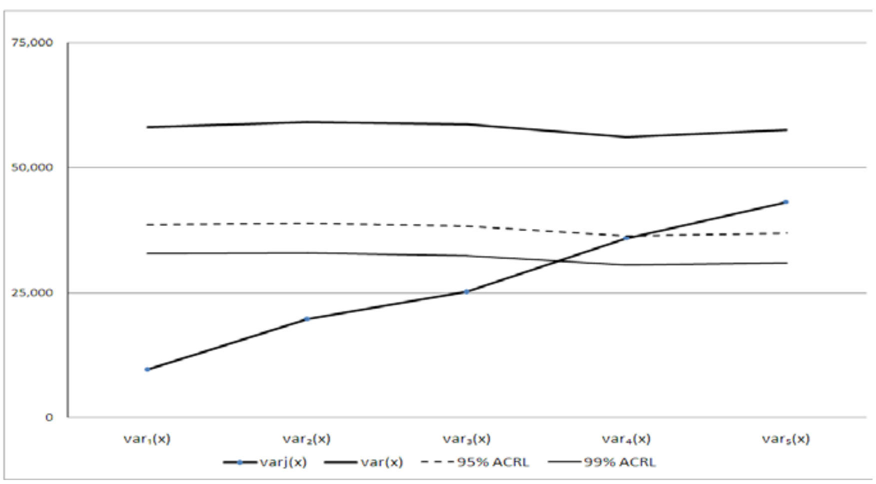

Figure B – Sampling variogram for silver

Lags of sampling variograms

A single line segment defined by a pair of variance terms intersects the lower limit of the asymmetric confidence range at 95% probability. The distance between this intersection and the Y-axis is l0.95 = j + {[(95% ACRL – $var_j(x)$] ÷ [$var_{j+1}(x)$ – $var_j(x)$]}, where 95% ACRL is the lower limit of the asymmetric 95% confidence range, $var_j(x)$ is the variance term below 95% ACRL, $var_{j+1}(x)$ is the variance term above 95% ACRL, and j is the whole number of variance terms below 95% ACRL. This distance is called the lag of the sampling variogram.

The sampling variogram for gold in Figure A shows that the line segment defined by $var_4(x)$ = 38.14 gpt^2 and $var_5(x)$ = 53.40 gpt^2 intersects 95% ACRL = 45.63 gpt^2 at l0.95 = 4 + {[(95% ACRL – $var_4(x)$] ÷ [$var_5(x)$ – $var_4(x)$]} = 4 + [(45.63 – 38.14) ÷ (53.40 – 38.14)] = 4.29 m. The measurement variance of var(spa) = 0.6891 gpt^2 is intrinsic to all variances in Table 5. Subtracting this measurement variance gives a lag of l0.95 = 4.30 m.

The sampling variogram for silver in Figure B shows that the line segment defined by $var_4(x)$ = 35,924 gpt^2 and $var_5(x)$ = 43,155 gpt^2 intersects 95% ACRL = 36,373 gpt^2 at l0.95 = j + {[95% ACRL –$var_4(x)$] ÷ [$var_5(x)$ – $var_4(x)$] = 4.06 m. Subtracting the measurement variance of var(spa) = 606.3 gpt^2 gives a lag of 4.09 m for the sampling variogram for silver.

Confidence limits for mass of contained metal

The variance of the mass of contained metal derives from the variance of a general function as defined in probability theory (Volk, 1980, Merks, 1985, Merks and Merks, 1989). In applied statistics, the unknown population mean μ is replaced with a sample mean $\bar{x}$ whereas the population variance σ^2 is replaced with var($\bar{x}$), the variance of a finite sample selected from a sampling unit or a sample space. The Central Limit Theorem defines the relationship between var(x), the variance of a sample, and var($\bar{x}$), the variance of its central value. The arithmetic mean is the central value of a set of measured values with equal weights. The central value of a set of measured values with variable weights is called a weighted average. Adjectives such as area-, count-, density-, mass- and volume- identify the weighted average under examination. Mass, length- and density-weighted averages are common in mineral exploration and mining.

Cerattepe's bulk sampling program gave arithmetic mean gold and silver grades. A density of 2.75 mt/m^3 was reported for this gossan ore. The sampling variogram for gold defines a mass of M = $\pi \cdot 4.30^2 \cdot 26 \cdot 2.75$ = 4,153 mt of excavated ore. The sampling variogram for silver defines a mass of M = $\pi \cdot 4.09^2 \cdot 26 \cdot 2.75$ = 3,760 mt of excavated ore. The mass of contained gold is Me(Au) = 4,153 · 10.86 ÷ 1000 = 45.09 kg. The mass of contained silver is Me(Ag) = 3,760 · 484.2 ÷ 1000 = 1,822 kg.

Table 7 - Confidence limits for masses of contained gold and silver

Statistic	Symbol	Gold	Silver
Mass of ore in mt	M	4,153	3,760
Metal grade in gpt	$\bar{x}$ (Me)	10.86	484.6
Mass of metal in kg	Me	45.09	1,822
Variance of metal mass	var(Me)	7.068	5,222
95% Confidence interval in kg	95% CI	5.34	145
95% Confidence interval in %rel	95% CI	11.8	8.0
95% Confidence range			
Lower limit in kg	95% CRL	39.7	1,677
Upper limit in kg	95% CRU	50.4	1,968

The confidence limits in Table 7 derive from the function Me = V · ID · $\bar{x}$ (Me), and its partial derivatives. In this case, the in-situ density of ID = 2.75 mt/m^3 is given as a constant. It is easy to prove that the partial derivative of volume adds significantly less to the variance of contained mass than does the partial derivative of grade. Therefore, the grade term adds most to the variance of the mass of contained metal. The grade term of $var(Me) = Me^2 \cdot [(CV_o \div 100)^2 \div n]$ is simple to derive (Merks, 2000). The volume term impacts little on the mass of contained metal. The in-situ density term should be taken into account when exploring massive sulfides.

Summary

Fisher's F-test verifies absence or presence of spatial dependence between in-situ ordered sets of independently measured values of stochastic variables such as gold and silver in the Cerattepe gossan ore deposit. The F-test is based on comparing the observed F-value between the variance of a set and the first variance term of the in-situ ordered set with tabulated F-values at 5%, 1% and 0.01% probability and with applicable degrees of freedom for each variance. If the observed F-value is highly significant, it makes sense to plot a sampling variogram. The line segment that intersects the lower limit of the asymmetric confidence range at 95% probability defines the distance between the Y-axis and the point where spatial dependence is statistically significant at 95% probability. Counting degrees of freedom is of critical importance when applying Fisher's test. Therefore, loss of degrees of freedom during reiteration should be taken into account.

A significant degree of spatial dependence gives a higher degree of precision for the central value of a set of independently measured values. Coefficients of variation are intuitive measures to check and compare degrees of variability at a glance whereas confidence intervals and ranges for central values are user-friendly measures for precision and risk. Confidence ranges for variances deserve as much attention and scientific scrutiny as do confidence ranges for central values.

Extrinsic measurement variances such as those associated with dividing whole core into halves, and with selecting and assaying large test portions for cyanide leaching, may be subtracted before Fisher's F-test is applied. The interleaved sampling protocol should be applied at bulk sampling stages in mineral exploration. Interleaved bulk samples give the least biased estimate for the measurement variance at affordable cost. Variances of dividing whole core into halves should be subtracted before the F-test is applied to obtain least biased estimates for intrinsic variances in ore deposits. A significant degree of spatial dependence between core samples within a borehole or between boreholes on a line gives unbiased confidence limits for masses of contained metals. In the latter case, a set of disjointed cylindrical volumes on a single line can be replaced with a set of contiguous blocks.

This method was applied in 1998 to test results for gold in core samples of variable length for a large set of boreholes drilled at a property of Barrick Gold Corporation. Permission will be asked to publish for teaching purposes a statistical analysis for a few lines of boreholes. Cominco Resources International Ltd granted permission in the early 1990s to publish a statistical analysis of test results for gold and silver determined in pairs of interleaved bulk samples selected at its Cerattepe exploration project in Turkey.

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