



The creator of Geostatistics, Professor Georges Matheron.



GEORGES MATHERON – FOUNDER OF SPATIAL STATISTICS

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Introduction

Professor Georges Matheron (1930-2000) made fundamental contributions to science by establishing new theoretical frameworks in spatial statistics, random sets, mathematical morphology and the physics of random media. In 1954 he commenced work on regionalized random variables inspired by De Wijs and Krige. The results were published in *Traité de Géostatistique Appliquée*: volume 1 (1962) on continuous spatial element concentration functions and volume 2 (1963) on linear estimators for local interpolation (kriging). The variogram was introduced as a tool to calculate block averages and their variances. Later additions by Matheron included the theory of “change of support” and nonlinear estimation (disjunctive kriging). These refinements were presented during the first geostatistical world congress in Rome (1975).

In 1968 Matheron had created the “*Centre de Géostatistique et de Morphologie Mathématique*” of the Paris School of Mines in Fontainebleau. Work on prediction of the hydrodynamic permeability of porous media in connection with the oil industry resulted in definition of basic operations for mathematical morphology. His book on “*Random Sets and Integral Geometry*” (1975) introduced these new concepts including Minkowski operations and the Boolean set. These methods of approach are used in various branches of physics and for image processing. Additionally, Matheron was interested in philosophy of science, especially in the problem of how data observed in part of the Earth’s crust could be the single realization of a random function with infinitely many possible realizations.

Georges Matheron was an eminent scientist whom I had the privilege of getting to know well between 1966 and 1983. Matheron spent most of his career at the Centre de Morphologie Mathématique in Fontainebleau. Although Georges Matheron disliked travel, I met him at scientific meetings in Lawrence, Montreal and Rome. I also visited him three times in Fontainebleau. Through this personal contact and correspondence Matheron helped me solve several problems in research on mineral resource evaluation (cf. Agterberg, 2001).

At a Memorial Meeting, Adrian Baddeley (2001) described Matheron’s life and scientific contributions. Dominique Jeulin (2000) wrote an obituary of Matheron listing his most prominent achievements and references. Matheron was a modest man who preferred to work in isolation documenting new results in the *Notes Géostatistiques* stored in the library of his institute in Fontainebleau. Visitors had free access to these notes and could keep copies for further study. Matheron inspired numerous other scientists first in France and later worldwide. His ideas and methods have been tested extensively and continue to be applied widely both inside and outside of geoscience.

Early Beginnings

Georges Matheron attended the *Ecole Polytechnique* and *Ecole Normale Supérieure des Mines de Paris*. He studied both geology and mathematics. In 1954 he began his career with the French Geological Survey in Algeria (BRGM). One of his first publications (Matheron, 1955a) concerns the Gara Djebilet oolitic iron deposit that had been discovered in 1952. This is a standard geological publication with folded geological map and descriptions of the stratigraphy, structure and genesis of this deposit of Early Devonian age. The final section on ore grades and tonnages provides estimates of the ore reserves. These estimates exceed 2 billion metric tons in total. They were obtained in a purely geometrical manner by measuring the shapes of contacts between formations from maps and cross-sections.

From his first publications it would seem that Matheron started out as a classical geologist. However, shortly afterwards he published a paper (Matheron, 1955b) on applications of statistical methods for ore reserve estimation that foreshadows the revolutionary approach to spatial statistics he was to bring about and that kept him in the forefront of research on spatial statistics during the last forty years of the 20th century. This first geostatistical paper is contained in a special issue of *Annales des Mines* on the use of

mathematical statistics in economic geology. It follows translations into the French of material published by Daniel G. Krige on ore reserve valuation of South African gold deposits.

The emphasis in this special issue and Matheron's contribution is on lognormal distribution of gold measures for samples, mining panels and blocks. Matheron emphasizes the "permanency" of lognormality in that gold assays from smaller and larger blocks all have lognormal frequency distributions with variances decreasing with increasing block size. He defines and discusses Krige's formula for the propagation of variances. This formula says that the variance for small blocks within a very large block is equal to the variance for the small blocks within intermediate-size blocks plus the variance of the intermediate-size blocks within the very large block (Matheron, 1955b, p. 59).

In order to explain permanency of lognormality, Matheron referred to a publication by De Wijs (1951). Before joining the Technical University of Delft in the Netherlands just after the Second World War, de Wijs had been working in Bolivia where, in isolation, he had developed the idea that, on average, the ratio of element concentration values in two halves of a block of rock remains constant, regardless of the size of the block. The fundamental constant controlling this ratio could be estimated from spatial series of element concentration values according to de Wijs (1948).

The formulas introduced by de Wijs differed drastically from those used by mathematical statisticians. I noticed this in 1958 when I was a graduate student at the University of Utrecht and had to conduct a seminar on the skew frequency distribution of ore assay values (later published in Agterberg, 1961). My criticism was that it would be better to apply the conventional method of serial correlation to series of assay values.

De Wijs disagreed with me and defended his model vigorously showing me publications including Matheron (1955b) where the model of de Wijs was further developed. Matheron's (1962) first book argued that conventional statistical approaches are not valid because the mathematical expectation (mean) of series of assay values does not necessarily exist. He introduced the intrinsic variogram as a new tool for dealing with these eventualities. This first book was a memoir of the French Geological Survey (BRGM).

In an English summary (Matheron, 1963a), a section on "Insufficiency of Classical Statistical Concepts" is followed by sections introducing "regionalized variables" and the variogram. Matheron argued that a mining assay cannot possibly be modeled as a random variable in the traditional statistical sense. He defined the regionalized variable as a function that assumes definite value $f(x)$ at every point x in three-dimensional space. The degree of continuity within the geometrical field V of the assay value $f(x)$ is represented by the semi-variogram:

$$\gamma(h) = \frac{1}{2V} \iiint_V [f(x+h) - f(x)]^2 dV$$

where h denotes distance between two points within V for which assay values are available. Matheron proposed to estimate the semi-variogram as half the average of squared differences between assay values that are distance h apart. This is permitted when the samples are very small with assay values that are good estimates of $f(x)$.

Four different types of variogram were defined. The first type has a parabolic trend at the origin and represents a regionalized variable with continuity such as bed-thickness. The second type is characterized by non-zero slope at the origin and represents a regionalized variable with "on average" continuity. This is the type most commonly encountered for assay values in ore deposits. The third type has a discontinuity or "nugget effect" at the origin. The fourth variogram is pure nugget effect showing a horizontal line only. It corresponds to the random variable of classical mathematical statistics.

Matheron (1962) almost exclusively deals with mathematical extensions of the model of de Wijs from which Matheron had derived a semi-variogram of the second type with:

$$\gamma(h) = 3\alpha \ln |h|$$

In this equation, α is a constant called "absolute dispersion". Matheron's chief argument for using the variogram instead of spatial covariance or serial correlation coefficients was that it is not possible to estimate these conventional statistics when the variance of $X = f(x)$ is infinitely large. Infinite variance is an

intrinsic property of the model of de Wijs. This also implies that the expected value $E(X) = \mu$ would not exist.

These arguments run counter to intuition because it seems reasonable to assume that an ore body has a mean grade that is equal to total amount of metal divided by volume. Today, the assumption of infinite variance is not as outrageous as it was in the past because of the more recent development of multifractal modeling and chaos theory. Perhaps Matheron was exposed to these kinds of ideas at university where he was a student of Paul Lévy now considered to be one of the fathers of chaos theory.

Matheron was not the only scientist to introduce the variogram. Independently, Jowett (1955) had introduced this same tool also for the reason that existence of a mean of a random variable should not be taken for granted. Although he was a meticulous scholar, Matheron essentially ceased referring to other scientists in a systematic manner during the early 1960s. It is likely that he was not aware of work by Jowett and others working along the same lines. Matheron's lack of attention in this respect did not endear him to some scientists. While working independently, Matheron made discoveries that included some results that were already in print (*cf.* Cressie, 1990; Baddeley, 2001).

During the 1960s I became gradually convinced of the validity of Matheron's approach. My book on "Geomathematics" (Agterberg, 1974) contains discussions of Matheron's geostatistical theory. If you know the equation of the variogram, you can calculate variances of element concentration values for blocks of rock of different sizes and shapes, as well as spatial covariances for pairs of such blocks situated at different spatial locations. This is possible because the variogram applies to point data. The variogram function can be integrated in many different ways. Matheron's (1962) first book is a *tour de force* in performing such calculations using the relatively simple logarithmic variogram resulting from the model of de Wijs.

When Matheron's geostatistical theory became more widely accepted and employed in practice, practitioners preferred not to use the logarithmic variogram with its infinite variance. The spherical variogram became the tool of choice. Frequently, a nested design consisting of several consecutive spherical variograms is used. The exponential variogram comes second in order of frequency of usage. Both spherical and exponential variogram have a "sill". For the spherical form this sill is equal to the variance at longer distances exceeding the "range". The exponential flattens gradually and its value asymptotically approaches the variance on the sill.

It is somewhat ironic that, contrary to the logarithmic variogram, both the spherical and the exponential imply finite variance with existence of a mean value. Thus the original argument for using the variogram instead of the autocorrelation function does not seem to be valid in practical applications. Matheron had initiated his development of geostatistics shortly after he had joined the French Geological Survey in 1954. The logarithmic variogram provided a good fit to the limited experimental data available to him at the time.

Kriging

Matheron honored Daniel Krige by coining the term "kriging". In Matheron (1967) he urges all scientists concerned with spatial interpolation to adopt this term that had been used routinely in France since 1960. The method first was explained in detail in Volume 2 of *Traité de Géostatistique Appliquée* called "Le krigéage" (Matheron (1963b). The original formulation is now known as "ordinary kriging" (see e.g. Journel and Huijbregts, 1978, p. 563). According to Cressie (1990, p. 244), it consists of three key components: (1) use of spatial covariances or variograms; (2) use of the best linear unbiased estimator (blue) for the mean μ instead of assuming μ is known or obtainable in some other way; and (3) use of spatial locations to define covariances (or semi-variograms) of the random process, rather than using covariances from a general multivariate-analysis setting. Cressie considered component (2) as defining kriging but remarks that Matheron in a letter to him felt that change-of-support plays a more important role (Cressie, 1990, p. 251).

Kriging evoked strong opposition from geologists and mining engineers. For example, Whitten (1966) preferred polynomial interpolation (trend-surface analysis). Matheron responded to this criticism in a paper on "Kriging, or Polynomial Interpolation Procedures?" subtitled: "A contribution to polemics in mathematical geology" (Matheron, 1967). It states that the kriging procedure is valid for stationary or intrinsic random functions, whereas the polynomial interpolation procedure should be applied only in

specific situations where an error $e(x)$ is superimposed on a regular phenomenon $m(x)$. Matheron presents an example of “how striking but fallacious evidence for a real trend may occur as a result of purely random cumulative effects”.

This example is based on a random-walk process model with stationary and stochastically independent increments rather than independent values. A process of this type has a variogram but it does not have a spatial covariance function. As far as I could see at that time (and now) this was not a good counterexample because, generally, a regular phenomenon $m(x)$ with superimposed residual $e(x)$ can be characterized by its spatial covariance function, and “simple kriging” with $m(x)$ as mean can be applied. In June 1970, Daniel Merriam organized a Colloquium on Geostatistics on Campus at the University of Kansas, Lawrence, sponsored by the Kansas Geological Survey. The speakers included Georges Matheron and John Tukey was the principal discussant.

I had been a visiting scientist at the Kansas Geological Survey in 1969/70 and the Colloquium provided an opportunity to present results obtained during my stay. I had tried to find out which technique is better: (a) trend surface analysis or (b) kriging. My paper (Agterberg, 1970) describes applications of these techniques to observed tops of the Arbuckle Formation in Kansas. Theoretically, kriging requires an autocorrelation function that is independent of location. Structural trends in the Arbuckle Formation would have to be eliminated before approximate stationarity could be achieved.

In order to test this hypothesis I used 200 subsurface observations. This data set was randomly divided into three subgroups. Two subsamples of 75 observations were used for control and a blind sample of 50 data was used to check predictions made by the first two subsamples. The results confirmed the hypothesis that trend surface analysis followed by kriging of the residuals provides better results than (a) trend surface analysis on its own, or (b) kriging of the original data. Tukey (1990) could agree with these results (also see Agterberg, 2001).

Matheron’s (1970a) paper at the Geostatistics Colloquium in Lawrence was on “Random Functions and their Application in Geology”. He presented basic concepts including the equations for the theoretical and estimated variogram. He referred to a paper by Huijbregts and Matheron (1970) that was to be presented in Montreal later that month at the 9th APCOM (Application of Computers and Operations Research in the Mineral Industries) Symposium. In that paper, the technique of universal kriging was introduced. In this approach, the element concentration of a block of rock has an expected value called “drift” that depends on geographic location. With respect to estimating this drift, Matheron (1970a, p.86) remarked: “the well-known problem of trend-surface analysis perhaps will encounter here its happy end”.

There was more discussion of trend surface analysis versus kriging in which Geoffrey Watson participated. Watson recently had succeeded Tukey as chair of the Statistics Department, Princeton University, and they were in agreement in their comments on this topic. They pointed out that trend surface analysis provides an unbiased but not necessarily optimal solution of the regression coefficients.

Matheron did not like to travel except within France. The previous year he had been an invited speaker at the 38th Session of the International Statistical Institute in London with William Krumbain as invited discussant. Unfortunately, Matheron sent a telegram to London just before his presentation was scheduled stating that he would not be able to come. His paper (Matheron, 1970b), however, was published. Krumbain was delighted to meet Matheron during the Lawrence Colloquium at a dinner party in Dan Merriam’s house. The guests were seated at separate tables, my wife next to Krumbain and Matheron. She was asked to translate all the questions that Krumbain had not been able to ask as discussant in London, and to translate Matheron’s answers back into English. This was no easy task.

Matheron argued that the concept of randomness expresses an epistemological decision that relations between the basic constituents of natural phenomena can be expressed as probabilistic. Not all probabilities can be determined experimentally. Pure empiricism is insufficient and probabilistic models should incorporate genetic ideas borrowed from geologists (see also Matheron, 1989).

Geostatistical Congress in Rome with follow-up during the late 1970s

In 1975 I participated in the two-week NATO Advanced Study Institute on “Advanced Mining Geostatistics” in Rome (cf. Agterberg, 1976). The Rome ASI is widely credited for making geostatistics

better known in the English speaking world. New methodology introduced in Rome by the French geostatistical school included nonlinear methods such as disjunctive kriging. I became particularly interested in the “discrete Gaussian model” (Matheron, 1976). This model can be employed to describe the change of frequency distributions of element concentration values associated with change of support. Several frequency distributions have the property of “permanency” in that they retain their shape regardless of size of block on which the average concentration value is determined.

As discussed before, the lognormal distribution has this property. If the frequency distribution of element concentration values for small blocks is lognormal, then the frequency distribution of larger blocks also is lognormal according to the discrete Gaussian model. Logarithmic variance decreases with increasing block size according to a relation determined by the spatial covariance function. This following approach to permanency is more general than the one based on the model of de Wijs.

The basic assumption underlying the discrete Gaussian model is that the expected value of a small block located within a larger block is equal to the value of the larger block, or

$$E(X_1 | X_2) = X_2$$

In this equation, E denotes mathematical expectation; X_1 and X_2 are random variables for element concentration values of the smaller and larger block, respectively. The gamma distribution and the negative binomial distribution also have the property of permanency.

My first paper (Agterberg, 1977) on this subject deals with proportions of felsic volcanics in 40km x40km cells in the Abitibi area. This proportion was assumed to satisfy a gamma distribution. Massive sulfide deposits can be related to felsic volcanics by means of a negative binomial distribution model. Parameters of the gamma and negative binomials for different cell sizes can be calculated from one another.

A serious problem I encountered in this approach is that the gamma distribution provides a poor model for the frequency distribution of proportion values when cell size is small. In very small cells, a rock type either is present or absent and its frequency density curve has the shape of a reversed letter J with its maximum at zero. I sent a copy of my paper to Matheron asking for input. This contact resulted in my second visit to Fontainebleau. Matheron (1974b) had solved my problem of the reversed J-shape frequency density function two years earlier. His report provided me with the material needed for statistical modeling described in Agterberg (1984).

Concluding Remarks

Georges Matheron preferred to be at his desk where he worked in full concentration without interruptions. Friendly to his coworkers, he could be opinionated and alienate others. In 1983 he was awarded the W.C. Krumbein medal of the International Association for Mathematical Geology but did not accept it. He felt that people should not be honored in this way during their lifetime.

There is worldwide consensus that Ronald Fisher and John Tukey belonged to the great mathematical statisticians of the 20th century. The position of Georges Matheron is more controversial. Scientists tend to be divided in their opinions. However, spatial statistics is used increasingly in many branches of science. The spread and popularizing of Matheron’s ideas have gone far beyond mining applications. For example, the program of the 53rd Session of the International Statistical Institute in Seoul, August 2001, had invited paper meetings on (a) spatial statistical modeling with environmental applications, (b) spatial statistics in an agricultural environment, and (c) statistics for spatial and spatial-temporal processes.

Matheron’s students and co-workers including Margaret Armstrong, Michel David, André Journel, Jean Serra, and Hans Wackernagel have greatly contributed to the dissemination of geostatistics. There now exist a large number of books on this topic including proceedings of the Geostatistical World Congresses held very four years. Because of his numerous outstanding contributions, Matheron deserves Fisher-Tukey class standing.

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