Borehole statistics with spreadsheet software

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Abstract

Spreadsheet software is used to convert test results for drill core samples into borehole statistics such as volumes and masses of in-situ ore, metal contents and variances of metal contents. The properties of variances and the sampling variogram play a pivotal role. The additive of property of variances gives a sound scientific basis to the calculation of confidence limits for metal contents and grades as a measure of the risks associated with estimating reserves and resources.

Introduction

Borehole statistics are simple to obtain with spreadsheet templates. The proper interpretation requires some understanding of sampling theory and practice. In mineral exploration, sampling is the process of selecting primary samples (drill core or cuttings), preparing test samples of primary samples and selecting and assaying test portions of test samples. The properties of variances form the basis for the calculation of unbiased confidence intervals and ranges for metal contents and grades as a measure for the risk associated with an intrinsically imprecise measurement chain.

Ordered sets of measured values may exhibit a significant degree of associative dependence and give a higher degree of precision for central values (arithmetic means or weighted averages) than randomly distributed sets do. For simplicity, associative dependence in n-dimensional sample spaces is referred to as spatial dependence. Downhole sampling variograms not only show where orderliness dissipates into randomness but also define cylindrical volumes and masses of in situ ore at 95% probability.

Preparing test samples of drill core or cuttings and selecting and assaying portions of test samples will add extraneous variance components to the intrinsic variability of the stochastic variable. Subtracting extraneous measurement variances from the variances of the randomly distributed and ordered sets of measured values increases the power of analysis of variance to detect spatial dependence and defines a larger cylindrical volume and mass of in situ ore. It also gives a higher degree of precision for the arithmetic mean (samples of identical length), for the length weighted average (samples of variable length) and for the length and density weighted average of test results for samples of variable length in massive sulfides.

Confidence limits for contents and grades are computed in the following stages:

- *Stage 1:* Estimate the radius of a cylindrical body from the downhole sampling variogram.
- *Stage 2:* Convert this volume into a mass based on the sampled length of the borehole and the in situ density of the ore.
- *Stage 3:* Calculate the metal content and the variance of metal content based on the borehole statistics.
- *Stage 4:* Sum masses, contents and variances of contents and convert these sums into confidence limits for the cumulative metal content and the mass weighted average grade.
- *Stage 5:* Determine whether disjointed cylinder bodies can be replaced with contiguous blocks by applying analysis of variance to the randomly distributed and ordered sets of weighted averages of ore zones within sections.
- *Stage 6:* Compute confidence limits for the cumulative content and the mass weighted average grade of all blocks.

All of these computations can be made with spreadsheet templates.

Sampling theory and practice

Sampling theory applies not only to a dynamic stochastic system, such as a mass of crushed ore during transfer, but also to a static stochastic system, such as a mass of in situ ore. Sampling practice deals with the tests and techniques required for estimating stochastic variables in an unbiased manner, irrespective of whether stochastic systems are static or dynamic.

The ratio between a primary sample mass and the mass it represents is referred to as the sampling ratio (Merks, 1985). Its dimension is a probability, and its reciprocal is the set of all primary samples into which a sampling unit can be divided. All primary samples in this set should have the same finite probability to be selected (equiprobable sample space). Under those conditions, each primary sample is unbiased, provided it is quantitatively extracted from the sampling unit and that it is prepared and analyzed in an unbiased manner. When crushed ore is sampled during transfer, the sampling ratio can

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Confidence limits for content

Table 6 gives the variances of gold contents based of the grade terms. This calculation reflects that the volume and density terms add only marginally and not significantly to the variance of content. The variance of the gold content of 0.72 kg for the first borehole is $var(Au) \approx Au^2*(CVa*0.01)^2*\sum wi^2 = 0.72^2*(80*0.01)^2*0.0658 = 0.022$ kg^2, and the variance of the gold content of 36.69 kg for the second borehole is $var(Au) - 36.69^2*(49*0.01)^2*0.0172 = 5.60$ kg^2.

The next step is to convert the variances of contents into confidence limits for each content and for their sum. Table 6 gives the 95% confidence intervals in g/t and % rel, the lower and upper limits of the symmetric 95% confidence range and the lower limit of the asymmetric 95% confidence range.

The additive property of contents and variances of contents implies that the variance of the sum of contents is the sum of the variances of contents. Thus, the variance of a gold content of $Au = 0.72 + 36.69 = 37.41$ kg is $var(Au) = 0.022 + 5.60 = 5.62$ kg^2. Evidently, the variance of the mass of contained gold derived from the statistics of the second borehole virtually determines the variance of the cumulative mass of contained gold.

For these boreholes, the sum of the variances of dividing whole core into halves, of preparing test samples of crushed halves and of selecting and assaying portions of the test samples is unknown. Logically, the sum of extraneous variances should be significantly lower than the variance of the randomly distributed set. After all, the intrinsic variability of gold in phantom deposits is statistically identical to zero.

Subtracting a measurement variance equivalent to a coefficient of variation of 40% from the variances of the randomly distributed and ordered sets increased the derived radii from 0.95 to 1.01 m for the first borehole and from 3.91 to 5.36 m for the second borehole. Therefore, a statistical quality-control (SQC) program should be implemented to monitor the measurement variance.

The question of whether disjointed cylindrical bodies can be replaced with contiguous blocks can be solved by determining the degree of spatial dependence between weighted average grades of ordered ore zones within sections. The variance between core sections within boreholes, of which the extraneous measurement variance is a component, may be subtracted from the variances of randomly distributed and ordered sets of weighted

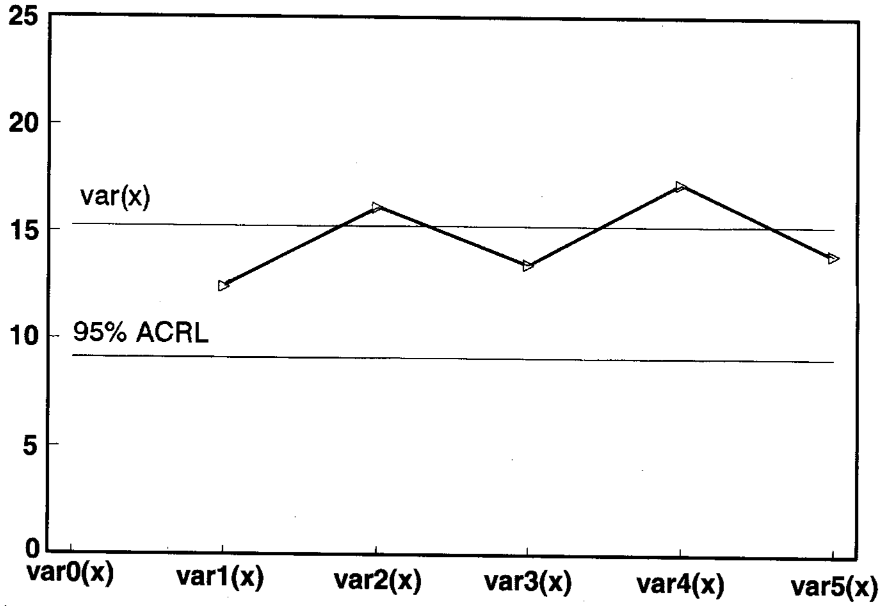

Figure 1 — Downhole sampling variogram for the first borehole (9801), in which the spacing between the variance terms for the ordered set is 1.30 m, the arithmetic mean for 16 core samples.

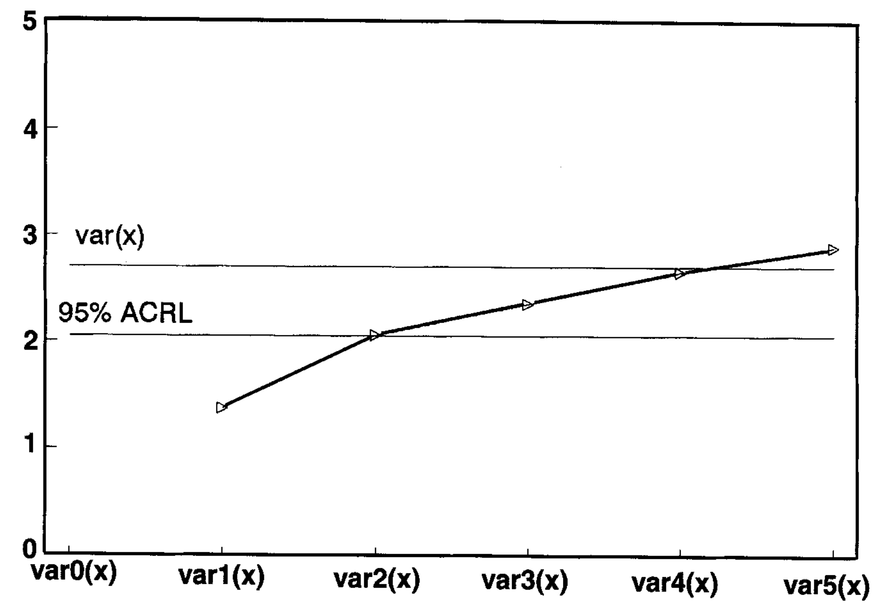

Figure 2 — Downhole sampling variogram for the second borehole (9802), in which the spacing between the variance terms for the ordered set is 1.98 m, the arithmetic mean for 59 core samples.

Table 6 — Confidence limits for gold content.

Statistic	Symbol	9801	9802	Sum
Gold content, kg	*Au*	0.72	36.69	37.41
Variance of gold content, kg^2	*var(Au)*	0.022	5.60	5.62
95 % Confidence interval, kg	95% *CI*	±0.30	±4.73	±4.74
95 % Confidence interval, % rel	95% *CI*	±41.2	±12.9	±12.7
Symmetric 95% confidence range:				
Lower limit in kg	95% *CRL*	0.42	32.0	32.7
Upper limit in kg	95% *CRU*	1.02	41.4	42.2
Asymmetric 95 % confidence range:	95% *ACR*			
Lower limit in kg	95% *ACRL*	0.48	32.8	33.5

Table 3 — Test for spatial dependence.

Statistic	Symbol	9801	9802
Variance of randomized set	*var(x)*	15.24	2.70
First term for ordered set	*var1(x)*	12.40	1.37
Calculated F-ratio	*F*	1.23	1.97
Significance	–	NS	**
Degrees of freedom for:			
Randomized set	*dfr*	15	58
Ordered set	*dfo*	30	116
Tabulated F-values at:			
95% probability	F0.95;*dfr*;*dfo*	2.02	1.44
99% probability	F0.99;*dfr*;*dfo*	2.70	1.61

NS = Not significant
** Significant at 99% probability

Table 4 — Statistics for sampling variograms.

Statistic	Symbol	9801	9802
Variance of randomized set	*var(x)*	15.24	2.70
Lower confidence limit	95% *ACRL*	9.12	2.05
Spacing, m	$\bar{l}$	1.30	1.98
Variances of ordered set:			
First term	*var1(x)*	12.40	1.37
Second term	*var2(x)*	16.13	2.06
Third term	*var3(x)*	13.46	2.35
Fourth term	*var4(x)*	17.20	2.65
Fifth term	*var5(x)*	13.92	2.89

Table 5 — Volume, mass and content defined by spatial dependence.

Statistic	Symbol	9801	9802
Weighted average grade	$\bar{x}w$	4.39	2.38
Sampled length, m	*l*	20.8	116.6
Derived diameter, m	*2r*	1.90	7.82
Gold content, kg/m	*ΔAu*	0.035	0.315
Volume, m^3	*V*	59	5,604
Mass*, mt	*M*	164	15,410
Gold content, kg	*Au*	0.72	36.69

*based on in-situ density of 2.75 mt/m^3

ing coefficients of variation do. Because its dimension is a percentage, the coefficient of variation is an intuitive measure for variability and precision.

Confidence limits for grades. Table 2 gives the confidence limits for the length weighted average grade of each borehole when reported as 95% confidence intervals in g/t and in relative percent, as lower and upper limits of symmetric 95% confidence ranges, and as lower limits of asymmetric 95% confidence ranges.

The variance of the length weighted average grade for core sections of variable length is obtained by multiplying the first variance term with $\sum wi^2$, the sum of the squared weighting factors. Similarly, the variance of the weighted average grade for core sections of variable length and density would be obtained by multiplying the first variance term with the sum of the squared weighting factors for density and length.

The 95% confidence intervals in Table 2 are 95% $CI = sd(\bar{x})*t0.95;df$ in g/t and 95% $CI = sd(\bar{x})*t0.95;df*100/\bar{x}$ in % rel. The lower and upper limits of the symmetric 95% confidence range are 95% $CRL = \bar{x}$ – 95% *CI* and 95% $CRU = \bar{x}$ + 95% *CI*. The lower limit of the asymmetric 95% confidence range is 95% $ACRL = \bar{x} - sd(\bar{x})*t0.90;df$.

Spatial dependence

The next step in the calculation of boreholes statistics is to determine whether the gold grades of ordered core samples within ore zones display a significant degree of spatial dependence by applying Fisher's F-test to the variance of the randomly distributed set and the first variance term for the ordered set. Table 3 gives the F-statistics for each borehole.

The calculated F-ratio of 15.24/12.40 ≈ 1.23 for the first borehole does not exceed the tabulated value of F0.95;15;30 = 2.02 at 95% probability, so that the ordered set does not display spatial dependence. By contrast, the F-ratio of 2.70/1.37 ≈ 1.97 for the second borehole exceeds F0.99;58; 116 = 1.61 at 99% probability, so that this ordered set displays a statistically significant degree of spatial dependence.

Sampling variogram

The sampling variogram for each borehole is obtained by plotting in a graph the first five variance terms for the ordered set, the variance of the randomly distributed set and the lower limit of the asymmetric 95% confidence range. The radius of the cylindrical body of in situ ore is the X-coordinate of the point where the sampling variogram intersects this lower limit. Table 4 gives the variances and the spacing between the variance terms for the ordered set.

Figure 1 displays the downhole sampling variogram for the first borehole in which the spacing between the variance terms for the ordered set is 1.30 m, the arithmetic mean for 16 core sections.

The first variance term of 12.40 $(g/t)^2$ for the ordered set exceeds the lower limit of 9.12 $(g/t)^2$ of the asymmetric 95% confidence range for the variance of 15.24 $(g/t)^2$ for the randomly distributed set. Hence, the grades of ordered core sections within the first borehole do not exhibit spatial dependence.

Figure 2 displays the downhole sampling variogram for the second borehole in which the spacing between the variance terms for the ordered set is 1.98 m, the arithmetic mean for 59 core sections.

The second variance term of 2.06 $(g/t)^2$ for the ordered set is virtually identical to the lower limit of 2.05 $(g/t)^2$ of the asymmetric 95% confidence range. Hence, the grades of ordered core sections within the second borehole display a significant degree of spatial dependence.

Volume, mass and content

Table 5 gives the volumes, masses and gold contents calculated from a radius of $r = [95\% \; ACRL/var1(x)]*\bar{l} = (9.12/12.40)*1.30 = 0.95$ m for the first borehole, and a radius of $r = \{[95\% \; ACRL - var1(x)]/[var2(x) - var1(x)] + 1\}*\bar{l} = [(2.05 - 1.37)/(2.06 - 1.37) + 1]*1.98 = 3.91$ m for the second borehole.

The boreholes give gold contents of 0.035 and 0.315 kg/m respectively. Apparently, infill drilling will yield less ore grade material in proximity to the high-grade borehole. More boreholes statistics would be required to determine whether high-grade boreholes do indeed define systematically less ore grade material than those with medium grades do.

where

$var(\bar{x})$ is the variance of central value,
wi is the weighting factor of i^{th} measured value and
$var(xi)$ is the variance of i^{th} measured value.

The arithmetic mean is the central value for a set of n measured values with identical weighting factors. Because each wi equals $1/n$, it follows that $\Sigma wi^2 = \Sigma(1/n)^2 = 1/n$ for a set of n measured values. By implication, the variance of the arithmetic mean of a set of n measured values determined in samples selected from the same population is $var(\bar{x}) = var(x)/n$. This elementary relationship is commonly referred to as the Central Limit Theorem.

Variance of metal content. The formula for the variance of metal content, or the mass of metal contained in a quantity of in situ ore, is derived from the variance of a general function as defined in probability theory (Merks and Merks, 1991). Following is a simplified formula for the variance of metal content

$$var(Me) = Me^2 * \left[var(V)/V^2 + var(D)/D^2 + var(GF)/GF^2\right] \quad (3)$$

where

$var(Me)$ is the variance of metal content,
Me is the metal content,
V is the volume in m^3,
D is the density in mt/m^3,
GF is the grade factor (dimensionless),
var(V) is the variance of volume in $(m^3)^2$
var(D) is the variance of density in $(mt/m^3)^2$ and
var(GF) is the variance of grade factor (dimensionless).

Invariably, the variance of grade adds most to the variance of content, so that the grade term $var(Me\,|\,GF) = Me^2 * var(GF)/GF^2$ is the dominant component of $var(Me)$. Given that the coefficient of variation is the standard deviation as a percentage of the measured value, or of the central value of a set, it can be shown that the variance of metal content is $var(Me) \approx Me^2 * (CVa * 0.01)^2 * \Sigma wi^2$, in which CVa is the coefficient of variation for the metal grade and Σwi^2 is the sum of the squared weighting factors. For example, if a set of sixty core sections of equal length defines a gold content of 25 kg, and if the variance of grade is equivalent to a coefficient of variation of 50%, the grade term of the variance of content is $var(Au\,|\,GF) \approx 25^2 * (50 * 0.01)^2 * 0.01667 = 2.60$ kg^2.

Confidence limits for variances. Variances, too, can only be estimated with a finite degree of precision. The lower limit of the asymmetric 95% confidence range for the variance of the randomized set of measured values and the variance terms for the ordered set are required to convert borehole statistics into volumes and masses of in situ ore. The lower limit of the asymmetric 95% confidence range for $var(x)$, the variance of a randomly distributed set of n measured values, is 95% $ACRL = var(x)/F0.95;df;\infty$ (Davies and Goldsmith, 1977; Volk, 1980; Merks, 1997). The values of $F0.95;df;\infty$ are listed in the F-distribution for 95% probability in the column for $df = n - 1$ degrees of freedom.

For example, a variance of 0.25 $(g/t)^2$, estimated from duplicate test results, has an asymmetric 95% confidence range with a lower limit of 95% $ACRL = var(x)/F0.95;1;\infty = 0.25/3.84 = 0.065$ $(g/t)^2$. When estimated from a set of 26 test results, the same variance would have an asymmetric 95% confidence range with a lower limit of 95% $ACRL = var(x)/F0.95;25;\infty = 0.25/1.51 = 0.166$ $(g/t)^2$.

Table 1 — Basic statistics for the two boreholes.

Statistic	Symbol	9801	9802
Arithmetic mean, g/t	$\bar{x}a$	4.24	2.33
Weighted average*, g/t	$\bar{x}w$	4.39	2.38
Randomized set:			
Variance	$var(x)$	15.24	2.70
Standard deviation	$sd(x)$	3.90	1.64
Coefficient of variation	CVr	89	69
Ordered set:			
First variance term	$var1(x)$	12.40	1.37
Standard deviation	$sd1(x)$	3.52	1.17
Coefficient of variation	CVo	80	49

*based on lengths of core sections

Table 2 — Confidence limits for weighted average grades for the two boreholes.

Statistic	Symbol	9801	9802
Weighted average grade, g/t	$\bar{x}w$	4.39	2.38
Sum of squared weighting factors	Σwi^2	0.0658	0.0172
95% confidence interval*, g/t	95% CI	±1.84	±0.30
95% confidence interval, % rel	95% CI	±42.1	±12.7
Symmetric 95% confidence range:			
Lower limit, g/t	95% CRL	2.54	2.08
Upper limit, g/t	95% CRU	6.23	2.68
Asymmetric 95% confidence range:	95% ACR		
Lower limit, g/t	95% $ACRL$	2.85	2.13

*Based on first variance terms for ordered sets

Borehole statistics

A single spreadsheet template is used to calculate the statistics for two boreholes drilled in the same gold deposit. It is simple to set up a template to calculate confidence limits for the cumulative gold content and mass weighted average grade of any number of boreholes. It is only marginally more complex to set up a template to test for spatial dependence between length weighted averages of ordered ore zones within sections. Regrettably, the large set of boreholes required to demonstrate this test are not available in the public domain.

Basic statistics. Test results for two sets of core sections of variable length are used to show how to calculate borehole statistics. Table 1 gives the basic statistics for a pair of boreholes drilled in the same gold deposit.

The arithmetic mean and the variance of the randomized set are obtained with spreadsheet functions. Separate columns are required to obtain the weighting factors for the length weighted average, and the squared weighting factors for the variance of this weighted average. Each variance term for the ordered set also requires two columns. The length weighted averages in Table 1 are used in all the calculations.

Table 1 shows that the variances between core samples within ore zones differ a great deal more than the correspondbe defined on an a priori basis. By contrast, the sampling ratio of in situ ore cannot be defined on an a priori basis.

Sampling ratios in mineral exploration are unique in the sense that the sampled length of a borehole, its downhole sampling variogram and the in situ density determine a mass of in situ ore and, thus, the sampling ratio between the primary sample mass (core or cuttings) and the mass it represents. Borehole statistics determine not only masses of in situ ore and contained metals (metal contents) but also confidence limits for metal contents and weighted-average grades. Confidence limits at 95% probability are commonly used in science and engineering, but lower probability levels may well be justified during early exploration stages when only a few boreholes and a limited number of test results for drill core or cuttings are available.

The process of selecting the bearing and azimuth of a borehole, of extracting drill core or cuttings, of preparing test portions of primary samples and of selecting and assaying test portions determine the borehole statistics. Each stage of the measurement chain has its own component of variance that becomes part of the variance of the randomly distributed set of measured values and of each variance term for the ordered set.

The primary sample-selection stage gives a measure for the intrinsic variability of the stochastic variable within its sample space. The sample preparation and analytical stages add extraneous variances to this intrinsic variability. The sum of the variances for the sample selecting, preparation and analytical stages can be partitioned into its components by applying analysis of variance. This simple test could have proved at an early stage that Bre-X's gold assays were fraudulent, because the intrinsic variability of gold in a phantom deposit is zero by definition and indeterminate in practice.

The variance of dividing whole core into halves adds a large component to the variance of grade and, thus, to the variance of content. Invariably, it adds significantly more to the variance of grade than the variances of subsequent preparation and analytical stages (Koch and Link, 1970; Merks 1985). A case can be made that statistical quality control be implemented to measure and monitor the variance of dividing whole core into halves, the variance of preparing test samples of crushed halves and the variance of selecting and assaying test portions of test samples. The Bre-X fraud underscores the devastating effect of tampering at the sample-preparation stage, the urgent need for statistical quality control in mineral exploration and the necessity to secure the integrity of whole core while stored on location and prior to division and comminution of retained halves and crushed halves (rejects).

The variance of selecting a test portion of a test sample is an integral component of the analytical variance. Therefore, a test sample should be properly homogenized to ensure that the selection of a test portion does not add too large a variance component. The variance of selecting and fire assaying test portions of Bre-X's salted test samples turned out to be truly astounding, because it is simply impossible to mix a little placer gold and a lot of crushed core! It is not surprising then that even the variance of selecting and cyanide leaching 750-g test portions was extraordinary high.

The single particle or nugget effect causes the variance of selecting test portions of test samples to be extremely high. Coefficients of variation of 1% to 2% were obtained in high-grade ore consisting of weathered sulfides with finely disseminated gold. Screening at 100-150 mesh and assaying coarse and fine fractions separately reduced the analytical variance in terms of a coefficient of variation for gold grades in the 10- to 30-g/t (0.3- to 0.9-oz/st) range from more than 50% to less than 5%, but it did so at a high cost.

The practice of cutting high gold grades cannot be justified in a scientific manner. Dixon's rejection criterion for outliers or rogue values should be applied to check whether a suspected value is an outlier or a valid member of the population (Volk, 1980). Dixon's criterion should be an integral part of SQC programs in mineral exploration.

Tampering at the analytical stage is primitive but prolific and is cloaked in creative semantics, pseudoscience and wishful thinking, which makes it much simpler to detect and avoid than fraud at the sample-preparation stage. The literature on fraudulent assay techniques is mandatory reading for those involved in mineral exploration (Bacon, Hawthorn and Poling, 1989).

Properties of variances

The variance is the basic measure for variability and precision. Variances are amenable to mathematical analysis. The additive property of variances within a measurement hierarchy and of variances of mass and content play essential roles in the calculation of unbiased confidence limits for contents and grades.

Variance of a randomly distributed set. One of the many functions in spreadsheet software gives the variance of a set of measured values. For example, the Lotus function @VARS gives the variance of a randomly distributed set of n measured values with identical weighting factors. In applied statistics, a set of n measured values has $df = n - 1$ degrees of freedom. Two columns in a spreadsheet template are required to calculate the variance of a randomly distributed set of measured values with variable weighting factors.

Variance terms for an ordered set. A randomly distributed set of n measured values has only one variance, but the ordered set has more variance terms. The following formula gives the variance terms for an ordered set of n measured values with variable weighting factors

$$var\,j(x) = \frac{\sum\left[w(i+j) * (xi+j-xi)^2\right]}{2(n-j)} \quad (1)$$

where

$varj(x)$ is the j^{th} variance term for the ordered set,
$xi + j$ is the $(i + j)^{th}$ measured value,
xi is the i^{th} measured value,
$wi + j$ is the distance weighting factor,
j is the spacing between the measured values,
n is the number of measured values for the j^{th} term and
$2(n - j)$ is the degrees of freedom for the j^{th} term.

Each term requires two columns in the spreadsheet template. Plotting the variance terms against their spacing gives a sampling variogram. Plotting the variance of the randomly distributed set and the lower limit of its asymmetric confidence range in the same sampling variogram gives a graphic interpretation of Fisher's F-test.

Variance of central value. The general formula for the variance of the central value (the arithmetic mean or a weighted average) of a set of n measured values is

$$var(\bar{x}) = \sum\left[wi^2 * var(xi)\right] \quad (2)$$

average grades within profiles before analysis of variance is applied. Tests for the absence or presence of anisotropy can also be applied at the final stage.

Summary

Spreadsheet software can be used to calculate unbiased confidence limits for gold contents and grades as a measure for the risk associated with estimating reserves and resources. Gold grades of ordered sets of core samples within ore zones determine the statistical fingerprints of boreholes. Analysis of variance shows whether spatial dependence is statistically significant and where orderliness dissipates into randomness. The central value of a set of measured values, the sampled length of a borehole, the downhole sampling variogram and the in situ density of the ore define the diameter of a cylindrical mass, its gold content and the variance of this gold content.

The cumulative mass of gold defined by a set of boreholes and its variance are obtained by summation and converted into confidence limits for content. The question of whether disjointed cylindrical bodies can be replaced with contiguous blocks is solved by applying analysis of variance to the weighted average grades of ordered boreholes within profiles. Only the first variance term for the ordered set is required to compute confidence limits for contents and weighted average grades of contiguous blocks. All the calculations can be implemented in a few spreadsheet templates.

Failures in recent years have proved that the risk associated with estimating grades and contents of gold deposits is extremely high. Applied statistics provides powerful tests and techniques to quantify those risks in terms of unbiased confidence limits for contents and grades and to test for bias at the sample selection, preparation and analytical stages of a complex and imprecise measurement chain. Implementing statistical quality control and maintaining the integrity of exploration samples at each stage of this measurement chain are of critical importance.

References

Bacon, W.G., Hawthorn, G.W., and Poling, G.W., 1989. "Gold analyses — myths, frauds and truths," *CIM Bulletin*, November.

Davies, O., and Goldsmith, P.L., 1977, *Statistical Methods in Research and Production*, Longman Group, London.

Koch, G.S., and Link, R.F., 1970, *Statistical Analysis of Geological Data*, John Wiley & Sons, New York.

Merks, J.W., 1985, *Sampling and weighing of Bulk Solids*, Trans Tech Publications, Clausthal-Zellerfeld.

Merks, J.W., and Merks, E.A.T., 1991, "Precision estimates for ore reserves," *Erzmetall*, October.

Merks, J.W., 1997, "Applied statistics in mineral exploration," *Mining Engineering*, February.

Volk, W., 1980, *Applied Statistics for Engineers*, Robert R. Krieger Publishing Co., Huntingdon.